Fully-Worked Solutions

PRACTICE 9

Formative Practice

1 **A** Correct

B Wrong

$y = \frac{1}{2}x + 2$

C Correct

D Correct

Answer: **B**

2 (a) (i) Gradient AB

$= \frac{5 - (-1)}{1 - (-2)}$

$= 2$

(ii) y-intercept of the straight line = 3

(b) (i) The gradient of the straight line is the coefficient of x of $y = 2x + 3$. ✓

(ii) The y-intercept of the straight line is the constant term of $y = 2x + 3$. ✓

(c) Equation of straight line $y = mx + c$ — Gradient: $\boxed{m}$; y-intercept: $\boxed{c}$

3

Equation of straight line	Gradient	y-intercept
(a) $y = 4x + 7$	4	7
(b) $y = -2x + 10$	–2	10
(c) $y = -x$	–1	0
(d) $y = 6$	0	6

4 (a) $-4y = -5x + 8$

$y = \frac{-5x + 8}{-4}$

$y = \frac{5}{4}x - 2$

(b) $\frac{y}{3} = -\frac{x}{10} + 1$

$y = -\frac{3}{10}x + 3$

(c) $\frac{7}{3}x - y = 7$

$\frac{\frac{7}{3}x - y}{7} = 1$

$\frac{x}{3} - \frac{y}{7} = 1$

5 (a) $4x - y = 8$

$y = 4x - 8$ [✓]

(b) $-4x + y = 8$

$y = 4x + 8$

(c) $-\frac{x}{2} + \frac{y}{8} = 1$

$-4x + y = 8$

$y = 4x + 8$

(d) $\frac{x}{2} - \frac{y}{8} = 1$

$4x - y = 8$

$y = 4x - 8$ [✓]

6

Straight line	Form $y = mx + c$	Gradient	y-intercept
(a) $-2x + 3y = 9$	$y = \frac{2}{3}x + 3$	$\frac{2}{3}$	3
(b) $\frac{x}{4} + \frac{y}{6} = 1$	$y = -\frac{3}{2}x + 6$	$-\frac{3}{2}$	6

7 (a) $x = 2, y = 5$

$4x - 3 = 4 \times 2 - 3$

$= 5$

$y = 4x - 3$

Point A that lies on the straight line $y = 4x - 3$ satisfies the equation of the straight line $y = 4x - 3$.

(b) $x = -1, y = -3$

$4x - 3 = 4 \times (-1) - 3$

$= -7$

$y \neq 4x - 3$

Point B that does not lie on the straight line $y = 4x - 3$ does not satisfy the equation of the straight line $y = 4x - 3$.

8 $x + 2y = 6$

$(0, 2)$: $x + 2y = 0 + 2(2)$

$= 4$

$\neq 6$

$\therefore$ (0, 2) does not lie on the straight line.

$(4, 1)$: $x + 2y = 4 + 2(1)$

$= 6$

$\therefore$ (4, 1) lies on the straight line.

$(-2, 4)$: $x + 2y = -2 + 2(4)$

$= 6$

$\therefore$ (−2, 4) lies on the straight line.

(2, 4): $x + 2y = 2 + 2(4)$
$= 10$
$\neq 6$

$\therefore$ (2, 4) does not lie on the straight line.

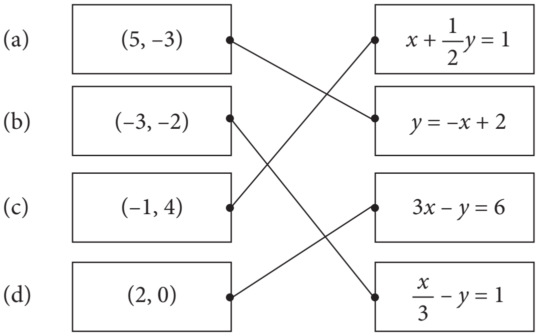
(12, −3): $x + 2y = 12 + 2(-3)$
$= 6$

$\therefore$ (12, −3) lies on the straight line.

(−3, 9): $x + 2y = -3 + 2(9)$
$= 15$
$\neq 6$

$\therefore$ (−3, 9) does not lie on the straight line.

9



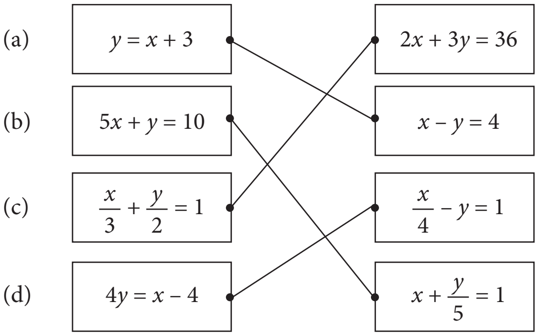
10

(a)

Straight line	Gradient
AB	$\frac{1}{2}$
CD	$\frac{1}{2}$
EF	$\frac{1}{2}$

(b) The gradients of parallel straight lines are equal.

11



12 (a) $y = 5$ (c) $y = -3$
(b) $x = 4$ (d) $x = -3$

13 (a) The equation of straight line is $y = 1$. ✓

(b) The equation of straight line is $x = 7$. ✓

(c) The equation of straight line is $y = 2x + 6$. ✗

(d) $y = -3x + c$
Substitute $x = 1$, $y = 0$,
$0 = -3(1) + c$
$c = 3$
The equation of straight line is $y = -3x + 3$. ✗

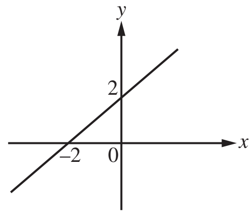
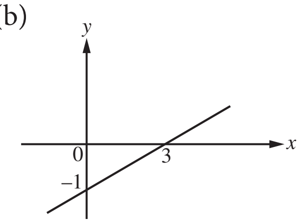
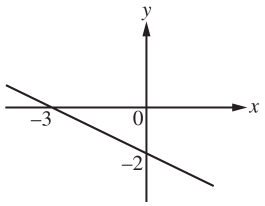
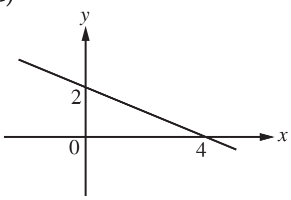
14 (a) $y = 4x + c$
Substitute $x = 3$, $y = 4$,
$4 = 4(3) + c$
$4 = 12 + c$
$c = -8$
The equation of straight line is $y = 4x - 8$.

(b) $y = -x + c$
Substitute $x = -1$, $y = 3$,
$3 = -(-1) + c$
$3 = 1 + c$
$c = 2$
The equation of straight line is $y = -x + 2$.

15 (a) $y = \frac{1}{2}x + c$
Substitute $x = 4$, $y = 0$,
$0 = \frac{1}{2}(4) + c$
$0 = 2 + c$
$c = -2$
The equation of the straight line k is $y = \frac{1}{2}x - 2$.

(b) $y = -2x + c$
Substitute $x = -1$, $y = 7$,
$7 = -2(-1) + c$
$7 = 2 + c$
$c = 5$
The equation of the straight line k is $y = -2x + 5$.

16

Straight line	x-intercept	y-intercept	Equation of straight line in the form $\frac{x}{a} + \frac{y}{b} = 1$
(a) 	-2	2	$-\frac{x}{2} + \frac{y}{2} = 1$
(b) 	3	-1	$\frac{x}{3} - y = 1$
(c) 	-3	-2	$-\frac{x}{3} - \frac{y}{2} = 1$
(d) 	4	2	$\frac{x}{4} + \frac{y}{2} = 1$

17 (a) $m = \frac{-6-2}{-1-3}$

$= \frac{-8}{-4}$

$= 2$

(b) (i) $2 = 2 \times 3 + c$

$2 = 6 + c$

$c = -4$

(ii) $-6 = 2 \times (-1) + c$

$-6 = -2 + c$

$c = -4$

18 (a) $m = \frac{1+3}{9-5}$

$= \frac{4}{4}$

$= 1$

$y = x + c$

Substitute $x = 9$, $y = 1$,

$1 = 9 + c$

$c = -8$

The equation of the straight line is $y = x - 8$.

(b) $m = \frac{8+7}{1+2}$

$= \frac{15}{3}$

$= 5$

$y = 5x + c$

Substitute $x = 1$, $y = 8$,

$8 = 5(1) + c$

$8 = 5 + c$

$c = 3$

The equation of the straight line is $y = 5x + 3$.

(c) $m = \dfrac{11 + 4}{-2 - 3}$

$= \dfrac{15}{-5}$

$= -3$

$y = -3x + c$

Substitute $x = -2$, $y = 11$,

$11 = -3(-2) + c$

$11 = 6 + c$

$c = 5$

The equation of the straight line is $y = -3x + 5$.

(d) $m = \dfrac{-1 + 3}{-4 - 4}$

$= \dfrac{2}{-8}$

$= -\dfrac{1}{4}$

$y = -\dfrac{1}{4}x + c$

Substitute $x = -4$, $y = -1$,

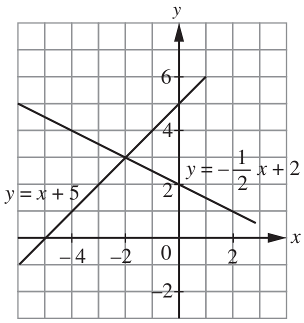
$-1 = -\dfrac{1}{4}(-4) + c$

$-1 = 1 + c$

$c = -2$

The equation of the straight line is $y = -\dfrac{1}{4}x - 2$.

19 (a)



(b) The coordinates of the point of intersection are $(-2, 3)$.

20 (a) $y = 4x - 5$ … ①

$y = 3$ … ②

Substitute ② into ①,

$3 = 4x - 5$

$4x = 8$

$x = 2$

$\therefore$ Point of intersection: $(2, 3)$

(b) $3x - 2y = 1$ … ①

$y = -5x - 7$ … ②

Substitute ② into ①,

$3x - 2(-5x - 7) = 1$

$3x + 10x + 14 = 1$

$13x = -13$

$x = -1$

From ②, $y = -5(-1) - 7$

$= 5 - 7$

$= -2$

$\therefore$ Point of intersection: $(-1, -2)$

(c) $y = x - 7$ … ①

$x - \dfrac{y}{4} = 4$ … ②

② × 4, $4x - y = 16$ … ③

① + ③, $4x = x + 9$

$3x = 9$

$x = 3$

From ①, $y = 3 - 7$

$= -4$

$\therefore$ Point of intersection: $(3, -4)$

(d) $2x + y = 1$ … ①

$-\dfrac{x}{12} + \dfrac{y}{6} = 1$ … ②

② × 24, $-2x + 4y = 24$ … ③

① + ③, $y + 4y = 1 + 24$

$5y = 25$

$y = 5$

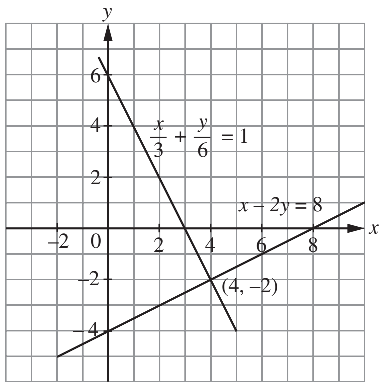
From ①, $2x + 5 = 1$

$2x = -4$

$x = -2$

$\therefore$ Point of intersection: $(-2, 5)$

21 (a)



Point of intersection: $(4, -2)$

(b) $x - 2y = 8$ … ①

$\dfrac{x}{3} + \dfrac{y}{6} = 1$ … ②

② × 6, $2x + y = 6$

$y = 6 - 2x$ … ③

Substitute ③ into ①,

$x - 2(6 - 2x) = 8$

$x - 12 + 4x = 8$

$5x = 20$

$x = 4$

From ③, $y = 6 - 2(4)$

$= -2$

Point of intersection: $(4, -2)$

(c) $x - 2y = 8 \ldots ①$

$\frac{x}{3} + \frac{y}{6} = 1 \ldots ②$

$② \times 12,\ 4x + 2y = 12 \ldots ③$

$① + ③,\ x + 4x = 8 + 12$

$5x = 20$

$x = 4$

From ①, $4 - 2y = 8$

$-2y = 4$

$y = -2$

Point of intersection: $(4, -2)$

22 (a) The y-intercept of the straight line BC is 4.

$\therefore B(0, 4)$

$4x + y = k$

Substitute $x = 0$, $y = 4$,

$4(0) + 4 = k$

$k = 4$

(b) The straight line AD is parallel to the x-axis.

$\therefore$ The equation of the straight line AD is $y = -2$.

(c) Gradient CD = Gradient AB

$= -4$

$y = -4x + c$

Substitute $x = 6$, $y = -2$,

$-2 = -4(6) + c$

$-2 = -24 + c$

$c = 22$

The equation of the straight line CD is $y = -4x + 22$.

(d) $4x + y = 4 \ldots ①$

$y = -2 \ldots ②$

Substitute ② into ①,

$4x - 2 = 4$

$4x = 6$

$x = \frac{3}{2}$

The coordinates of point A are $\left(\frac{3}{2}, -2\right)$.

Summative Practice

1 A $3y = x + 12$

When $x = 0$,

$3y = 12$

$y = 4$

The y-intercept = 4

B $\frac{1}{2}y = 2x - 3$

When $x = 0$,

$\frac{1}{2}y = -3$

$y = -6$

The y-intercept = -6

C $-\frac{x}{6} + \frac{y}{3} = 1$

When $x = 0$,

$\frac{y}{3} = 1$

$y = 3$

The y-intercept = 3

D $5x - 3y = 9$

When $x = 0$,

$-3y = 9$

$y = -3$

The y-intercept = -3

Answer: **C**

2 $3y = kx + 5$

$y = \frac{k}{3}x + \frac{5}{3}$

Gradient = $\frac{k}{3}$

$x + y = 10$

$y = -x + 10$

Gradient = -1

$\frac{k}{3} = \frac{1}{2} \times (-1)$

$k = -\frac{3}{2}$

Answer: **B**

3 Gradient,

$m = -\frac{8}{(-4)}$

$= 2$

y-intercept,

$c = 8$

The equation of the straight line is $y = 2x + 8$.

Answer: **A**

4 $\frac{k - 5}{-8 - 2} = \frac{4}{5}$

$\frac{k - 5}{-10} = \frac{4}{5}$

$k - 5 = -8$

$k = -3$

Answer: **B**

5 $px + qy = 6$

Substitute $x = 1$, $y = -3$,

$p(1) + q(-3) = 6$

$p - 3q = 6 \ldots ①$

Substitute $x = -2$, $y = -12$,

$p(-2) + q(-12) = 6$

$-2p - 12q = 6$

$-p - 6q = 3 \ldots ②$

$① + ②,\ -3q - 6q = 9$

$-9q = 9$

$q = -1$

From ①, $p - 3(-1) = 6$
$p + 3 = 6$
$p = 3$

Alternative method

Gradient, $m = \dfrac{-12 + 3}{-2 - 1}$
$= \dfrac{-9}{-3}$
$= 3$

$y = 3x + c$
Substitute $x = 1$, $y = -3$,
$-3 = 3(1) + c$
$-3 = 3 + c$
$c = -6$
The equation of the straight line is $y = 3x - 6$ or $3x - y = 6$.
$\therefore p = 3, q = -1$

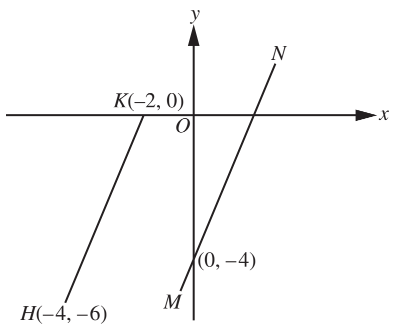
Answer: **D**

6 $qy = (2p + 5)x + 9$

(a) Straight line that is parallel to the x-axis has equation of the form $y = k$.
$2p + 5 = 0$ and $q \neq 0$
$2p = -5$
$p = -\dfrac{5}{2}$
$\therefore q \neq 0, p = -\dfrac{5}{2}$

(b) Straight line that is parallel to the y-axis has equation of the form $x = h$.
$q = 0$ and $2p + 5 \neq 0$
$2p \neq -5$
$p \neq -\dfrac{5}{2}$
$\therefore p \neq -\dfrac{5}{2}, q = 0$

7



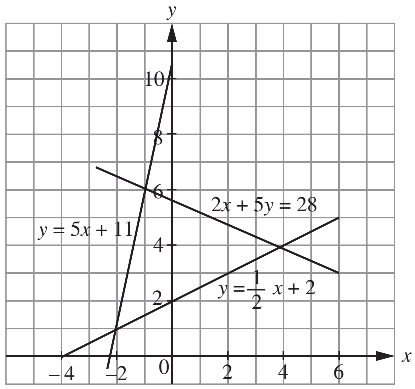
Gradient MN = Gradient HK
$= \dfrac{0 + 6}{-2 + 4}$
$= \dfrac{6}{2}$
$= 3$
$m = 3$
y-intercept, $c = -4$
The equation of the straight line MN is $y = 3x - 4$.

8 (a) (i) $\dfrac{x}{2} + \dfrac{y}{(-4)} = 1$
$\dfrac{x}{2} - \dfrac{y}{4} = 1$

(ii) Gradient of the straight line l
$m = -\dfrac{-4}{2}$
$= 2$
$y = 2x + c$
Substitute $x = -3$, $y = 0$,
$0 = 2(-3) + c$
$0 = -6 + c$
$c = 6$
The equation of the straight line l is $y = 2x + 6$ or $2x - y = -6$.

(b) Straight line k:
$\dfrac{x}{2} - \dfrac{y}{4} = \dfrac{4}{2} - \dfrac{14}{4}$
$= 2 - \dfrac{7}{2}$
$= -\dfrac{3}{2}$
$\neq 1$
$\therefore$ Straight line k does not pass through point (4, 14).
Straight line l:
$2x - y = 2(4) - 14$
$= 8 - 14$
$= -6$
$\therefore$ Straight line l passes through point (4, 14).

9 (a)

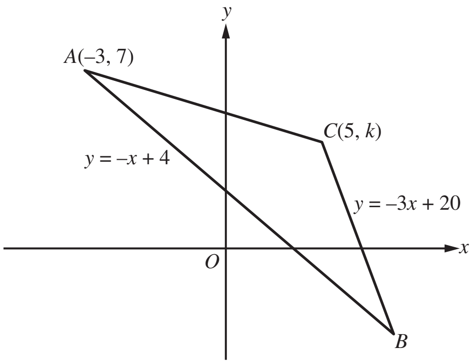


The points of intersection of the straight lines are (–2, 1), (4, 4) and (–1, 6).

(b) $y = 5x + 11$ … ①
$2x + 5y = 28$ … ②
Substitute $y = 5x + 11$ into ②,

$2x + 5(5x + 11) = 28$
$2x + 25x + 55 = 28$
$27x = 28$
$x = -1$
From ①, $y = 5(-1) + 11$
$= 6$
The point of intersection $(-1, 6)$ of the straight lines $y = 5x + 11$ and $2x + 5y = 28$ is the same as the graphical method.

10



(a) $y = -3x + 20$
Substitute $x = 5$, $y = k$,
$k = -3(5)+ 20$
$= -15 + 20$
$= 5$

(b) $y = -3x +20$ … ①
$x + y = 4$ … ②
Substitute ① into ②,
$x + (-3x + 20) = 4$
$-2x + 20 = 4$
$-2x = -16$
$x = 8$
From ①, $y = -3(8) + 20$
$= -24 + 20$
$= -4$
The coordinates of B are $(8, -4)$.

(c) Let the straight line AC cuts the x-axis at point $E(a, 0)$.
Gradient AE = Gradient AC
$\frac{7-0}{-3-a} = \frac{7-5}{-3-5}$
$\frac{7}{-3-a} = \frac{2}{-8}$
$-28 = -3 - a$
$-25 = -a$
$a = 25$
The x-intercept of the straight line AC is 25.

11 (a) RS is parallel to the y-axis.
x-coordinate of point S = x-coordinate of point R
$a = -4$

(b) (i) The equation of the straight line PQ is $y = -3$.
(ii) The equation of the straight line RS is $x = -4$.

(c) (i) Gradient PS
$= \frac{1+3}{-4-2}$
$= -\frac{4}{6}$
$= -\frac{2}{3}$
$y = -\frac{2}{3}x + c$
Substitute $x = 2$, $y = -3$,
$-3 = -\frac{2}{3}(2) + c$
$-3 = -\frac{4}{3} + c$
$c = -3 + \frac{4}{3}$
$= -\frac{5}{3}$
$y = -\frac{2}{3}x - \frac{5}{3}$
$3y = -2x - 5$
$2x + 3y = -5$
The equation of the straight line PS is $2x + 3y = -5$.

(ii) Gradient QR
$= -\frac{2}{3}$
$y = -\frac{2}{3}x + c$
Substitute $x = -4$, $y = 3$,
$3 = -\frac{2}{3}(-4) + c$
$3 = \frac{8}{3} + c$
$c = 3 - \frac{8}{3}$
$= \frac{1}{3}$
$y = -\frac{2}{3}x + \frac{1}{3}$
$3y = -2x + 1$
$2x + 3y = 1$
The equation of the straight line QR is $2x + 3y = 1$.

12 (a) $OQ = 2$ units
$\therefore Q(0, -2)$
The equation of the straight line QR is $y = -2$.

(b) Gradient PQ
$= \frac{3+2}{-2-0}$
$= -\frac{5}{2}$
$y = -\frac{5}{2}x - c$

Substitute $x = -2$, $y = 3$,

$3 = -\frac{5}{2}(-2) + c$

$3 = 5 + c$

$c = -2$

The equation of the straight line PQ is $y = -\frac{5}{2}x - 2$.

(c) QR is parallel to the x-axis.
$QR = 5$ units
$\therefore R(5, -2)$

$y = -\frac{5}{2}x + c$

Substitute $x = 5$, $y = -2$,

$-2 = -\frac{5}{2}(5) + c$

$-2 = -\frac{25}{2} + c$

$c = \frac{21}{2}$

$y = -\frac{5}{2}x + \frac{21}{2}$

When $y = 0$,

$-\frac{5}{2}x + \frac{21}{2} = 0$

$\frac{5}{2}x = \frac{21}{2}$

$x = \frac{21}{5}$

The x-intercept of the straight line RS is $\frac{21}{5}$.

Alternative method

Let the straight line RS cuts the x-axis at point $T(a, 0)$.

Gradient RT = Gradient RS

$\frac{0 + 2}{a - 5} = -\frac{5}{2}$

$4 = -5(a - 5)$

$4 = -5a + 25$

$5a = 21$

$a = \frac{21}{5}$

The x-intercept of the straight line RS is $\frac{21}{5}$.

(d) (i) $y = \frac{5}{3}x + c$

Substitute $x = 5$, $y = -2$,

$-2 = \frac{5}{3}(5) + c$

$-2 = \frac{25}{3} + c$

$c = -\frac{31}{3}$

$y = \frac{5}{3}x - \frac{31}{3}$

$3y = 5x - 31$

$5x - 3y = 31$

The equation of the straight line is $5x - 3y = 31$.

(ii) $5x - 3y = 31 \ldots ①$

$y = -\frac{5}{2}x - 2$

$2y = -5x - 4$

$5x + 2y = -4 \ldots ②$

② – ①, $5y = -35$

$y = -7$

From ①, $5x - 3(-7) = 31$

$5x + 21 = 31$

$5x = 10$

$x = 2$

The point of intersection of the two straight lines is $(2, -7)$.