Penyelesaian Lengkap

PRAKTIS 4

Kertas 1

Bahagian A

1 (a)
$$\begin{aligned}81(3^{x+7}) &= 3^{x-1}\times 9^x\\ 3^4(3^{x+7}) &= 3^{x-1}\times(3^2)^x\\ 3^{4+x+7} &= 3^{x-1}\times 3^{2x}\\ 3^{x+11} &= 3^{x-1+2x}\\ 3^{x+11} &= 3^{3x-1}\end{aligned}$$

Bandingkan indeks/*Compare index*,
$$\begin{aligned}x+11 &= 3x-1\\ 12 &= 2x\\ x &= 6\end{aligned}$$

(b) $2^m = 3^p = 1.5^w$
$$\begin{aligned}2^m = 3^p &\Rightarrow 2 = 3^{\frac{p}{m}}\\ 1.5^w &= 3^p\\ \left(\frac{3}{2}\right)^w &= 3^p\\ \left(\frac{3}{\left(3^{\frac{p}{m}}\right)}\right)^w &= 3^p\\ \left(3^{1-\frac{p}{m}}\right)^w &= 3^p\\ 3^{1-\frac{p}{m}} &= 3^{\frac{p}{w}}\\ \frac{m-p}{m} &= \frac{p}{w}\\ w &= \frac{mp}{m-p}\end{aligned}$$

2 (a)
$$\begin{aligned}3^{w-1}+3^w &= 324\\ \frac{3^w}{3}+3^w &= 324\\ \frac{4}{3}(3^w) &= 324\\ 3^w &= 324\times\frac{3}{4}\\ 3^w &= 243\\ 3^w &= 3^5\\ w &= 5\end{aligned}$$

(b)
$$\begin{aligned}4^{x+3}-4^{x+2}+2(4^x) &= 1\,600\\ 64(4^x)-16(4^x)+2(4^x) &= 1\,600\\ 50(4^x) &= 1\,600\\ 4^x &= 32\\ 2^{2x} &= 2^5\end{aligned}$$

Bandingkan indeks/*Compare index,*
$$\begin{aligned}2x &= 5\\ x &= 2.5\end{aligned}$$

3 (a)
$$\begin{aligned}3^{x+3}-3^{x+2}-3^{x+1} &= \frac{5}{9}\\ 27(3^x)-9(3^x)-3(3^x) &= \frac{5}{9}\\ 15(3^x) &= \frac{5}{9}\\ 3^x &= \frac{1}{27}\\ 3^x &= 3^{-3}\end{aligned}$$

Bandingkan indeks/*Compare index*,
$x = -3$

(b)
$$\begin{aligned}9^x &= 3^x+72\\ (3^2)^x &= 3^x+72\\ 3^{2x} &= 3^x+72\\ 3^{2x}-3^x-72 &= 0\\ (3^x)^2-3^x-72 &= 0\end{aligned}$$

Katakan/*Let* $u = 3^x$,
$$\begin{aligned}u^2-u-72 &= 0\\ (u+8)(u-9) &= 0\\ u+8=0,\ u-9 &= 0\\ u=-8,\quad u &= 9\end{aligned}$$

Apabila $3^x = -8$, nilai x tidak wujud.
Apabila $3^x = 9$, $x = 2$
When $3^x = -8$, *the value of* x *does not exist.*
When $3^x = 9$, $x = 2$

4 (a) $(5-\sqrt{6})^2$
$$\begin{aligned}&= (5-\sqrt{6})(5-\sqrt{6})\\ &= 25-10\sqrt{6}+(\sqrt{6})^2\\ &= 25-10\sqrt{6}+6\\ &= 31-10\sqrt{6}\end{aligned}$$

(b) $(2+3\sqrt{2})(3\sqrt{2}-2)$
$$\begin{aligned}&= (3\sqrt{2}+2)(3\sqrt{2}-2)\\ &= (3\sqrt{2})^2-2^2\\ &= 3^2\times 2-2^2\\ &= 18-4\\ &= 14\end{aligned}$$

5 (a) $\frac{1}{\sqrt{5}-2}-3$
$$\begin{aligned}&= \frac{1}{\sqrt{5}-2}\times\frac{\sqrt{5}+2}{\sqrt{5}+2}-3\\ &= \frac{\sqrt{5}+2}{5-4}-3\\ &= \sqrt{5}+2-3\\ &= \sqrt{5}-1\end{aligned}$$

(b) $\frac{1-\sqrt{3}}{2+\sqrt{3}}-\frac{\sqrt{3}-1}{\sqrt{3}+1}$

$= \frac{1-\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} - \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$

$= \frac{2-\sqrt{3}-2\sqrt{3}+3}{4-3} - \frac{3-\sqrt{3}-\sqrt{3}+1}{3-1}$

$= 5-3\sqrt{3} - \frac{4-2\sqrt{3}}{2}$

$= 5-3\sqrt{3}-2+\sqrt{3}$

$= 3-2\sqrt{3}$

6 (a) $(\sqrt{5})x = 8 + x$

$(\sqrt{5})x - x = 8$

$(\sqrt{5}-1)x = 8$

$x = \frac{8}{(\sqrt{5}-1)}$

$= \frac{8}{(\sqrt{5}-1)} \times \frac{\sqrt{5}+1}{\sqrt{5}+1}$

$= \frac{8\sqrt{5}+8}{5-1}$

$= \frac{8(\sqrt{5}+1)}{4}$

$= 2(\sqrt{5}+1)$

$\therefore m = 2, n = 1$

(b) $x - 8\sqrt{x} + 15 = 0$

$(\sqrt{x}-3)(\sqrt{x}-5) = 0$

$\sqrt{x}-3 = 0, \quad \sqrt{x}-5 = 0$

$\sqrt{x} = 3, \quad \sqrt{x} = 5$

$x = 9, \quad x = 25$

7 (a) $\log_2 224 - \log_2 7$

$= \log_2 \frac{224}{7}$

$= \log_2 32$

$= \log_2 2^5$

$= 5 \log_2 2$

$= 5(1)$

$= 5$

(b) $\log_3 16 - \log_3 36 - \log_3 \frac{108}{27}$

$= \log_3 \left(\frac{16}{36} \times \frac{27}{108}\right)$

$= \log_3 \left(\frac{1}{9}\right)$

$= \log_3 1 - \log_3 9$

$= \log_3 1 - \log_3 3^2$

$= 0 - 2$

$= -2$

8 $\log_5 \left(\frac{a}{b}\right) = 2 + 4\log_5 a - 2\log_5 b$

$\log_5 a - \log_5 b = 2 + 4\log_5 a - 2\log_5 b$

$-3\log_5 a + \log_5 b = 2$

$-\log_5 a^3 + \log_5 b = 2$

$\log_5 \left(\frac{b}{a^3}\right) = 2$

$\frac{b}{a^3} = 5^2$

$b = 25a^3$

9 $\log_3 \left[\ln\left(1+\frac{3}{y}\right)^2\right] = x$

$\ln\left(1+\frac{3}{y}\right)^2 = 3^x$

$2\ln\left(1+\frac{3}{y}\right) = 3^x$

$\ln\left(1+\frac{3}{y}\right) = \frac{3^x}{2}$

$1+\frac{3}{y} = e^{\frac{3^x}{2}}$

$\frac{3}{y} = e^{\frac{3^x}{2}} - 1$

$y = \frac{3}{e^{\frac{3^x}{2}} - 1}$

10 (a) $3(2^x) = 27$

$2^x = 9$

$x = \frac{\log_{10} 9}{\log_{10} 2}$

$= \frac{0.9542}{0.3010}$

$= 3.17$

(b) $\log_e \left(\frac{8e^6}{81}\right) = \log_e 8e^6 - \log_e 81$

$= \log_e 8 + 6\log_e e - \log_e 81$

$= \log_e 2^3 + 6\log_e e - \log_e 3^4$

$= 3m + 6 - 4k$

11 $\log_4 (3x+1) = 1 + \log_4 7$

$\log_4 (3x+1) - \log_4 7 = 1$

$\log_4 \left(\frac{3x+1}{7}\right) = 1$

$\frac{3x+1}{7} = 4$

$3x+1 = 28$

$3x = 27$

$x = 9$

12 $\log_3 (2x-3) + \log_3 5 = 2$

$\log_3 5(2x-3) = 2$

$5(2x-3) = 3^2$

$10x - 15 = 9$

$10x = 24$

$x = \frac{12}{5}$

13 Luas/*Area* $= AD \times AB$

$(4\sqrt{13}+8) = (\sqrt{13}-1) \times AB$

$AB = \frac{4\sqrt{13}+8}{\sqrt{13}-1} \times \frac{\sqrt{13}+1}{\sqrt{13}+1}$

$= \frac{4(13)+4\sqrt{13}+8\sqrt{13}+8}{13-1}$

$= \frac{60+12\sqrt{13}}{12}$

$= 5+\sqrt{13}$

$\therefore AB = (5+\sqrt{13})$ m

14 $58\,000 \times 0.92^n < 30\,000$

$0.92^n < 0.5172$

$$\log_{10} 0.92^n < \log_{10} 0.5172$$
$$n \log_{10} 0.92 < \log_{10} 0.5172$$
$$n(-0.0362) < -0.2863$$
$$n > 7.9088$$
$$\therefore n = 8$$

Pada akhir tahun ke-8.
At the end of 8th year.

Bahagian B

15 (a) $\frac{6}{3+\sqrt{3}} = \frac{6}{3+\sqrt{3}} \times \frac{3-\sqrt{3}}{3-\sqrt{3}}$
$$= \frac{6(3-\sqrt{3})}{3^2-(\sqrt{3})^2}$$
$$= \frac{18-6\sqrt{3}}{9-3}$$
$$= \frac{18-6\sqrt{3}}{6}$$
$$= 3-\sqrt{3}$$

(b) $2x + 24 = 14\sqrt{x}$
$$2x - 14\sqrt{x} + 24 = 0$$
$$x - 7\sqrt{x} + 12 = 0$$
$$(\sqrt{x}-3)(\sqrt{x}-4) = 0$$
$$\sqrt{x}-3 = 0, \sqrt{x}-4 = 0$$
$$\sqrt{x} = 3, \quad \sqrt{x} = 4$$
$$x = 9, \quad x = 16$$

(c) $\log_3 (3x+6) = 2 + \log_3 2x$
$$\log_3 (3x+6) - \log_3 2x = 2$$
$$\log_3 \left(\frac{3x+6}{2x}\right) = 2$$
$$\frac{3x+6}{2x} = 3^2$$
$$\frac{3x+6}{2x} = 9$$
$$3x + 6 = 18x$$
$$15x = 6$$
$$x = \frac{2}{5}$$

16 (a) $3^{2x} = 8.4$
$$2x = \frac{\log_{10} 8.4}{\log_{10} 3}$$
$$2x = \frac{0.9243}{0.4771}$$
$$2x = 1.9373$$
$$x = 0.9687$$

(b) $\log_9 p = \log_3 4$
$$\frac{\log_3 p}{\log_3 3^2} = \log_3 4$$
$$\frac{\log_3 p}{2} = \log_3 4$$
$$\log_3 p = 2 \log_3 4$$
$$\log_3 p = \log_3 16$$
$$\therefore p = 16$$

(c) $\log_2 \sqrt{x-5} = \log_4 8$
$$\log_2 (x-5)^{\frac{1}{2}} = \frac{\log_2 2^3}{\log_2 2^2}$$
$$\frac{1}{2}\log_2 (x-5) = \frac{3}{2}$$
$$\log_2 (x-5) = 3$$
$$x - 5 = 8$$
$$x = 13$$

17 (a) $\log_x 8 = \frac{\log_{\sqrt{x}} 8}{\log_{\sqrt{x}} x}$
$$= \frac{\log_{\sqrt{x}} 8}{\frac{\log_x x}{\log_x \sqrt{x}}}$$
$$= \frac{\log_{\sqrt{x}} 8}{2}$$
$$= \frac{1}{2}\log_{\sqrt{x}} 8$$
$$= \log_{\sqrt{x}} \sqrt{8}$$
$$\therefore \quad N = \sqrt{8}$$

(b) $\log_3 (2x-3) = \log_9 (3-x)$
$$\log_3 (2x-3) = \frac{\log_3 (3-x)}{\log_3 3^2}$$
$$2 \log_3 (2x-3) = \log_3 (3-x)$$
$$\log_3 (2x-3)^2 = \log_3 (3-x)$$
$$(2x-3)^2 = 3-x$$
$$4x^2 - 12x + 9 + x - 3 = 0$$
$$4x^2 - 11x + 6 = 0$$
$$(x-2)(4x-3) = 0$$
$$x - 2 = 0, 4x - 3 = 0$$
$$x = 2, \quad x = \frac{3}{4}$$

(c) $\log_8 (3x+6) = \log_2 3$
$$\frac{\log_2 (3x+6)}{\log_2 2^3} = \log_2 3$$
$$\log_2 (3x+6) = 3 \log_2 3$$
$$\log_2 (3x+6) = \log_2 27$$
$$3x + 6 = 27$$
$$3x = 21$$
$$x = 7$$

18 (a) (i) $\log_3 1.4 = \log_3 \left(\frac{7}{5}\right)$
$$= \log_3 7 - \log_3 5$$
$$= q - p$$

(ii) $\log_9 9.8 = \frac{\log_3 9.8}{\log_3 9}$
$$= \frac{\log_3 \left(\frac{49}{5}\right)}{\log_3 3^2}$$
$$= \frac{\log_3 7^2 - \log_3 5}{2}$$
$$= \frac{2q-p}{2}$$
$$= q - \frac{p}{2}$$

(b) $\log_{25} m - \log_5 n = \frac{1}{2}$
$$\frac{\log_5 m}{\log_5 5^2} - \log_5 n = \frac{1}{2}$$

$$\frac{\log_5 m}{2} - \log_5 n = \frac{1}{2}$$
$$\log_5 m - 2\log_5 n = 1$$
$$\log_5\left(\frac{m}{n^2}\right) = 1$$
$$\frac{m}{n^2} = 5$$
$$m = 5n^2$$

Kertas 2

Bahagian A

1 (a)
$$\log_5 p = \frac{3}{\log_{np} 5}$$
$$\log_5 p = \frac{3}{\left(\frac{\log_5 5}{\log_5 np}\right)}$$
$$\log_5 p = \frac{3}{\left(\frac{1}{\log_5 np}\right)}$$
$$\log_5 p = 3\log_5 np$$
$$\log_5 p = 3(\log_5 n + \log_5 p)$$
$$\log_5 p = 3\log_5 n + 3\log_5 p$$
$$2\log_5 p = -3\log_5 n$$
$$\log_5 p = -\frac{3}{2}\log_5 n$$
$$p = n^{-\frac{3}{2}}$$

(b)
$$\log_u 125 - \log_{\sqrt{u}} 5u + 1 = 0$$
$$\log_u 125 - \frac{\log_u 5u}{\log_u \sqrt{u}} = -1$$
$$\log_u 125 - \frac{\log_u 5u}{\frac{1}{2}} = -1$$
$$\log_u 125 - 2\log_u 5u = -1$$
$$\log_u 125 - \log_u 25u^2 = -1$$
$$\log_u\left(\frac{125}{25u^2}\right) = -1$$
$$\frac{125}{25u^2} = u^{-1}$$
$$\frac{5}{u^2} = \frac{1}{u}$$
$$u = 5$$

2 (a)
$$2\log_3(p-3) = 1 + 4\log_3 p + \log_3 pq$$
$$\log_3(p-3)^2 = \log_3 3 + 4\log_3 p + (\log_3 p + \log_3 q)$$
$$\log_3(p-3)^2 = \log_3 3 + 5\log_3 p + \log_3 q$$
$$\log_3(p-3)^2 = \log_3(3p^5q)$$
$$(p-3)^2 = 3p^5q$$
$$q = \frac{(p-3)^2}{3p^5}$$

(b)
$$5^{2x} = 5^{x+1} - 6$$
$$(5^x)^2 - 5(5^x) + 6 = 0$$
Katakan/*Let* $u = 5^x$
$$u^2 - 5u + 6 = 0$$
$$(u-2)(u-3) = 0$$
$$u - 2 = 0,\ u - 3 = 0$$
$$u = 2,\quad u = 3$$
Apabila/*When* $5^x = 2$,
$$x = \frac{\log_{10} 2}{\log_{10} 5} = 0.4307$$
Apabila/*When* $5^x = 3$,
$$x = \frac{\log_{10} 3}{\log_{10} 5} = 0.6826$$

3
$$\log_2(6x+5) - 2\log_8 x^6 + 3\log_2 x$$
$$= \log_2(6x+5) - 12\log_8 x + 3\log_2 x$$
$$= \log_2(6x+5) - 12\left(\frac{\log_2 x}{\log_2 2^3}\right) + 3\log_2 x$$
$$= \log_2(6x+5) - 12\left(\frac{\log_2 x}{3}\right) + 3\log_2 x$$
$$= \log_2(6x+5) - 4\log_2 x + 3\log_2 x$$
$$= \log_2(6x+5) - \log_2 x$$
$$= \log_2\left(\frac{6x+5}{x}\right)$$
$$\log_2(6x+5) - 2\log_8 x^6 + 3\log_2 x = 4$$
$$\log_2\left(\frac{6x+5}{x}\right) = 4$$
$$\frac{6x+5}{x} = 2^4$$
$$6x + 5 = 16x$$
$$10x = 5$$
$$x = \frac{1}{2}$$

4 (a) Katakan/*Let* $\log_N M = x$
$$\log_N M = x$$
$$M = N^x$$
$$\log_u M = x\log_u N$$
$$\log_u M = \log_u N^x$$
$$x = \frac{\log_u M}{\log_u N}$$
$$\therefore \log_N M = \frac{\log_u M}{\log_u N}$$

(b)
$$\log_4 \frac{128x^2}{y^3}$$
$$= \frac{\log_2\left(\frac{128x^2}{y^3}\right)}{\log_2 2^2}$$
$$= \frac{\log_2 2^7 + \log_2 x^2 - \log_2 y^3}{2}$$
$$= \frac{7 + 2p - 3q}{2}$$

Bahagian B

5 (a) (i)
$$\log_2 3.5 = \log_2 \frac{7}{2}$$
$$= \log_2 7 - \log_2 2$$
$$= w - 1$$

(ii)
$$\log_8 756 = \frac{\log_2 756}{\log_2 8}$$

$$= \frac{\log_2 (4 \times 7 \times 27)}{3}$$
$$= \frac{1}{3}(\log_2 2^2 + \log_2 7 + \log_2 3^3)$$
$$= \frac{1}{3}(2 + w + 3u)$$
$$= \frac{2}{3} + \frac{1}{3}w + u$$

(b) $2^x \times 2^{3y} = 32$
$2^x \times 2^{3y} = 2^5$
$x + 3y = 5 \ldots$ ①
$\frac{3^x}{3^{2y}} = 3$
$3^{x-2y} = 3$
$x - 2y = 1 \ldots$ ②
① – ②: $5y = 4$
$y = \frac{4}{5}$

Daripada/*From* ①,
$x + 3\left(\frac{4}{5}\right) = 5$
$x = 5 - \frac{12}{5}$
$= \frac{13}{5}$

6 (a) (i) $\log_n 0.75 = \log_n \frac{3}{2^2}$
$= \log_n 3 - 2 \log_n 2$
$= q - 2p$

(ii) $\log_6 24 = \frac{\log_n 24}{\log_n 6}$
$= \frac{\log_n (2^3 \times 3)}{\log_n (2 \times 3)}$
$= \frac{3 \log_n 2 + \log_n 3}{\log_n 2 + \log_n 3}$
$= \frac{3p + q}{p + q}$

(b) $\log_2 x^4 y = 5$
$4 \log_2 x + \log_2 y = 5 \ldots$ ①
$\log_2 \left(\frac{x^3}{y}\right) = 3$
$3 \log_2 x - \log_2 y = 3 \ldots$ ②
① + ②: $7 \log_2 x = 8$
$\log_2 x = 1.143$
$x = 2^{1.143} = 2.208$

Daripada/*From* ①,
$4(1.143) + \log_2 y = 5$
$\log_2 y = 0.428$
$y = 2^{0.428} = 1.345$
$\therefore x = 2.208, y = 1.345$