Fully-Worked Solutions

FORM 4

CHAPTER 6 Linear Law

Self Test 1

1 (a) $m = \dfrac{9-4}{1-6}$

$= -\dfrac{5}{5}$

$= -1$

$Y = mx + c$

$(1, 9), \quad 9 = (-1)(1) + c$

$c = 10$

$\therefore \log y = -\dfrac{1}{x} + 10$

(b) (i) $y = 8, \log 8 = -\dfrac{1}{x} + 10$

$\dfrac{1}{x} = 10 - \log 8$

$x = \dfrac{1}{10 - \log 8}$

$x = 0.11$

(ii) $x = \dfrac{5}{17}, \log y = -\dfrac{1}{\frac{5}{17}} + 10$

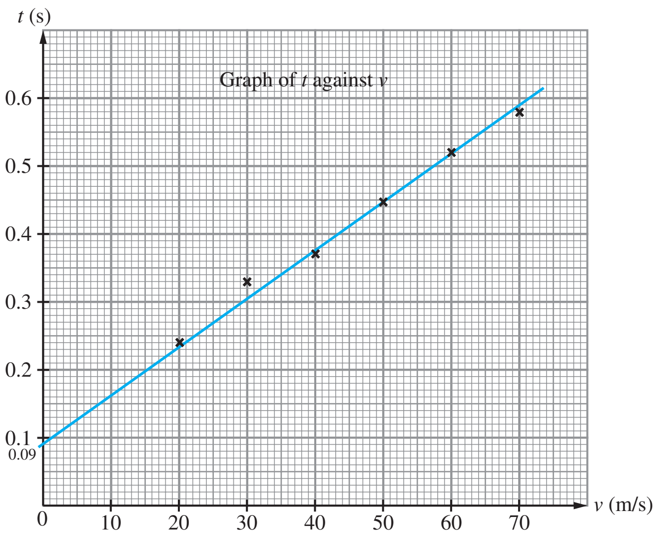
$= -\dfrac{17}{5} + 10$

$= \dfrac{33}{5}$

$y = 10^{\frac{33}{5}}$

$y = 3\,981\,071.71$

2 (a)



(b) $m = \dfrac{0.52 - 0.24}{60 - 20}$

$= 0.007$

From the graph, y-intercept $= c = 0.9$

$\therefore t = 0.007v + 0.9$

(c) From the graph, when $t = 0.4$ s, $v = 43$ m/s

Self Test 2

1 (a) $y = \dfrac{p}{x-q}$

$y(x-q) = p$

$xy - yq = p$

$x - q = \dfrac{p}{y}$

$\dfrac{1}{y} = \dfrac{x}{p} - \dfrac{q}{p}$

$\therefore Y = \dfrac{1}{y}, m = \dfrac{1}{p}, X = x, c = -\dfrac{q}{p}$

(b) $py = q^x$

$\lg py = \lg q^x$

$\lg p + \lg y = x \lg q$

$\lg y = x \lg q - \lg p$

$\therefore Y = \lg y, m = \lg q, X = x, c = -\lg p$

2 (a) $m = \dfrac{12-0}{2-(-4)} = 2$

$(2, 12), 12 = 2(2) + c$

$c = 8$

$\therefore xy = 2x + 8$

$y = \dfrac{8}{x} + 2$

(b) (i) $y = 7, 7 = \dfrac{8}{x} + 2$

$x = \dfrac{8}{5}$

(ii) $x = 10.5, y = \dfrac{8}{10.5} + 2$

$y = \dfrac{58}{21}$

3 (a) $m = \dfrac{-8-0}{2-6} = 2$

$(6, 0), 0 = 2(6) + c$

$c = -12$

$\therefore y^2 = 2(x-1) - 12$

(b) $y^2 = 2(x-1) - 12$

$= 2x - 2 - 12$

$= 2x - 14$

$y = \sqrt{2(x-7)}$

(c) $y^2 = 9$

$2x - 14 = 9$

$2x = 23$

$x = \dfrac{23}{2}$

4 $y = 5 - 3x^2, Y = 5X + c$

$y = 5 - 3x^2$

$\dfrac{y}{x^2} = \dfrac{5}{x^2} - 3$

$\dfrac{y}{x^2} = 5\left(\dfrac{1}{x^2}\right) - 3$

where

$Y = \dfrac{y}{x^2}, X = \dfrac{1}{x^2}, m = 5$

$\therefore c = -3$

Self Test 3

1 $\frac{x}{2p}+\frac{y^2}{3q}=1$

$\frac{y^2}{3q}=1-\frac{x}{2p}$

$y^2=-\frac{3q}{2p}x+3q$

y^2-intercept $=9$

$3q=9$

$\therefore q=3$

$-\frac{3q}{2p}=-\frac{3}{2}$

$q=p$

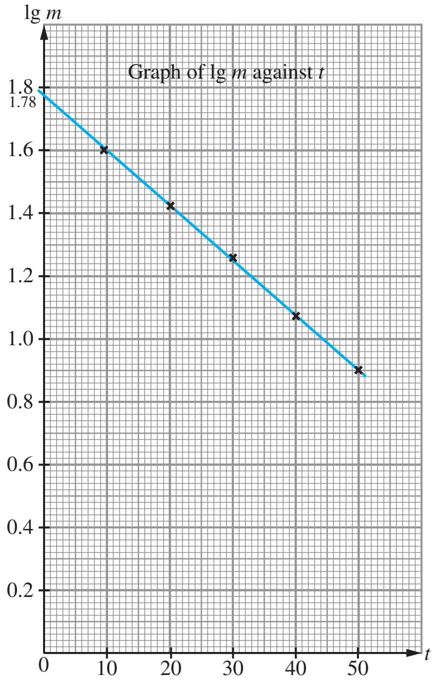
$\therefore p=3$

2 (a) $m=q\left(10^{-kt}\right)$

$\lg m=\lg q+\lg 10^{-kt}$

$\lg m=-kt+\lg q$

t	10	20	30	40	50
lg *m*	1.60	1.43	1.26	1.09	0.91



$m=\frac{1.6-0.98}{10-45}=-0.018$

$c=1.78$

Thus, $-k=-0.018$ $\qquad$ $\lg q=1.78$

$k=0.018$ $\qquad$ $q=10^{1.78}=60.26$

(b) When $t=28$, $\lg m=1.28$

$m=10^{1.28}$

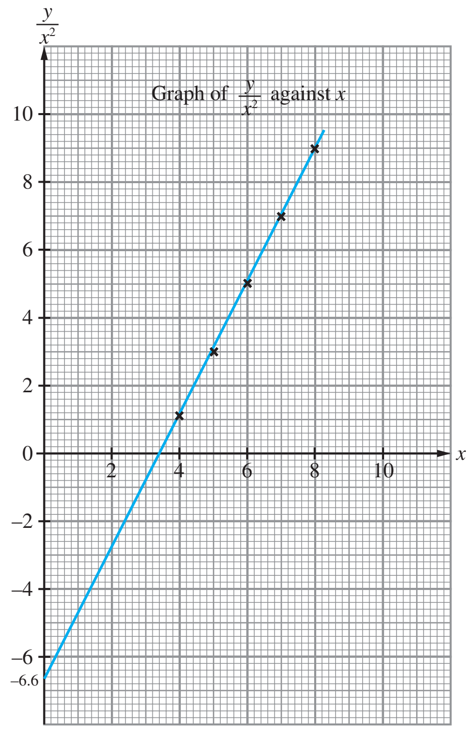
$=19.1$ g

3 $y=ax^3+bx^2$

$\frac{y}{x^2}=ax+b$

x	4	5	6	7	8
$\frac{y}{x^2}$	1.125	3	5	6.94	9.06

(a)



(b) (i) Gradient, $m=\frac{1.12-6.94}{4-7}$

$=1.95$

y-intercept, $c=-6.6$

$\therefore a=1.95, b=-6.6$

(ii) $x=5.5, \frac{y}{x^2}=4$

$y=4(5.5)^2$

$=121$

SPM Practice

Paper 1

1 $y=2x^4+5x^2$

$\frac{y}{x^2}=2x^2+5 \Rightarrow m=2, c=5$

(a) $A=$ point of the y-intercept

$\therefore A(0, 5)$

(b) $m=\frac{p-5}{4-0}$

$2=\frac{p-5}{4}$

$p-5=8$

$p=13$

(c) $x^2=6, \left(\frac{y}{x^2}\right)=2(6)+5$

$=17$

$y=17(6)$

$y=102$

2 $y=a\sqrt{x}+\frac{b}{\sqrt{x}}$

$y\sqrt{x}=ax+b$

$m=a=\frac{3-0}{0-8}$

$a=-\frac{3}{8}$

y-intercept, $c=3$

$\therefore b=3$

3 $ax^2 - by^2 = 1$

$by^2 = ax^2 - 1$

$y = \frac{a}{b}x^2 - \frac{1}{b}$

$m = \frac{a}{b} = \frac{4}{9}$

$a = \frac{4}{9}b$

$(27, 8),\ 8 = \frac{4}{9}(27) + c$

$c = -4$

$c = -\frac{1}{b}$

$-4 = -\frac{1}{b}$

$b = \frac{1}{4}$

$a = \frac{4}{9}\left(\frac{1}{4}\right) = \frac{1}{9}$

4 $4x = qy - 2px^2$

$qy = 4x + 2px^2$

$\frac{y}{x} = \frac{4}{q} + \frac{2p}{q}x$

$m = \frac{2p}{q}$

$3 = \frac{2p}{q}$

$2p = 3q$ .... ①

$(-2, 0),\ 0 = c + 3(-2)$

$c = 6$

$\therefore \frac{4}{q} = 6$

$q = \frac{2}{3}$

From ①, $2p = 3\left(\frac{2}{3}\right)$

$p = 1$

5 $Y = mX + c$

$Y = \log_{10} y, X = \log_{10} x$

$m = \frac{8 - 4}{4 - 2} = 2$

$(2, 4), 4 = 2(2) + c$

$c = 0$

$\therefore \log_{10} y = 2\log_{10} x$

$y = x^2$

6 $Y = mX + c$

(a) $Y = xy^2, X = \frac{1}{x}$

$m = \frac{8 - 0}{3 - (-1)} = 2$

$(-1, 0), 0 = 2(-1) + c$

$c = 2$

$\therefore Y = 2X + 2$

$xy^2 = 2\left(\frac{1}{x}\right) + 2$

$y^2 = \frac{2}{x^2} + \frac{2}{x}$

$y = \sqrt{\frac{2 + 2x}{x^2}}$

$y = \frac{\sqrt{2 + 2x}}{x}$

(b) (i) $y^2 = \frac{2}{x^2} + \frac{2}{x}$

$9 = \frac{2}{x^2} + \frac{2}{x}$

$9x^2 = 2 + 2x$

$9x^2 - 2x - 2 = 0$

$x = -0.373, x = 0.595$

(ii) $x = 3, y^2 = \frac{2}{9} + \frac{2}{3}$

$= \frac{8}{9}$

$y = \pm\sqrt{\frac{8}{9}}$

$y = \pm\frac{2}{3}\sqrt{2}$

7 $y = x\left(ax - 1 + \frac{b}{x}\right)$

$y = ax^2 - x + b$

$y + x = ax^2 + b$

(a) $m = \frac{6 - (-2)}{8 - 0} = 1$

$c = -2$

$\therefore Y = X - 2$

$m = a = 1$

$c = b = -2$

(b) $x = 15, y + 15 = 1(15)^2 - 2$

$y = 208$

8 $\left(y = 2 + \frac{3y}{x}\right) \div y$

$1 = \frac{2}{y} + \frac{3}{x}$

$\frac{2}{y} = -\frac{3}{x} + 1$

$\frac{1}{y} = -\frac{3}{2x} + \frac{1}{2}$

$Y = \frac{1}{y}, m = -\frac{3}{2}, X = \frac{1}{x}, c = \frac{1}{2}$

$(1, r), r = -\frac{3}{2}(1) + \frac{1}{2}$

$r = -1$

$(s, 2),\ 2 = -\frac{3}{2}(s) + \frac{1}{2}$

$\frac{3}{2} = -\frac{3}{2}s$

$s = -1$

Paper 2

1 $P = a\,(27)kt$

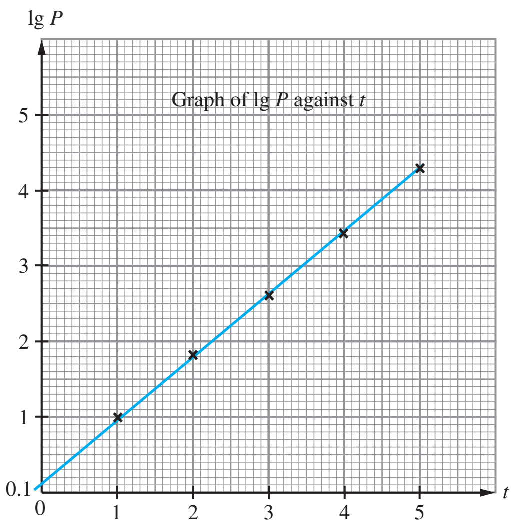
(a) $\lg P = \lg a + \lg 27^{kt}$

$= (k\lg 27)\,t + \lg a$

The straight line is obtained by plotting $\lg P$ against t where $Y = \lg P$, $X = t$, $m = k\lg 27$ and $c = \lg a$.

(b)

t	1	2	3	4	5
$\lg P$	0.97	1.80	2.62	3.45	4.27



(c) $m = \dfrac{3.45 - 0.97}{4 - 1}$

$(\lg 27)\ k = 0.827$

$k = 0.58$

$c = \lg a$

$0.1 = \lg a$

$a = 10^{0.1}$

$a = 1.26$

2 (a) $\dfrac{x}{p} + \dfrac{y^2}{q} = 1$

$\dfrac{y^2}{q} = -\dfrac{1}{p}x + 1$

$y^2 = -\dfrac{q}{p}x + q$

(i) $m = -\dfrac{q}{p}$

$\dfrac{9 - 4}{0 - (-2)} = -\dfrac{q}{p}$

$\dfrac{q}{p} = -\dfrac{5}{2}$

$p = -\dfrac{2}{5}q$ .... ①

y-intercept, $c = 9$

$\therefore q = 9$

From ①, $p = -\dfrac{2}{5}(9)$

$p = -\dfrac{18}{5}$

(ii) $Y = mX + c$

$Y = \dfrac{5}{2}(2) + 9$

$= 14$

$y^2 = 14$

$y = \sqrt{14}$

(b) $y = ax^n$

$\lg y = \lg a + n \lg x$

(i) y-intercept, $= 0$

$\lg a = 0$

$a = 10^0 = 1$

$(2, 4),\ 4 = 2n + 0$

$n = 2$

(ii) $(p, 9),\ \lg y = 2 \lg x$

$2p = 9$

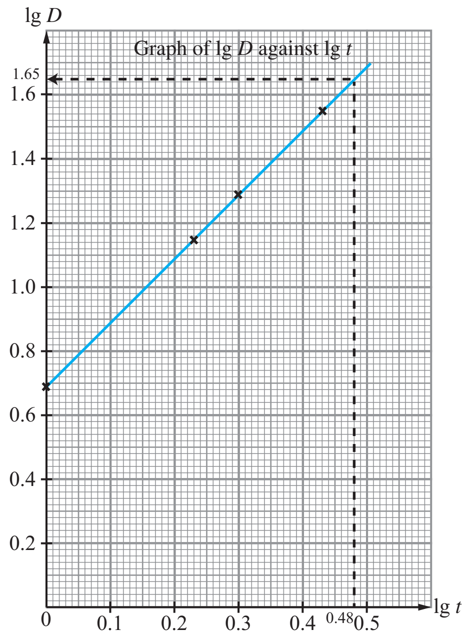
$p = \dfrac{9}{2}$

3 (a) $D = a \times t^b$

$\lg D = \lg a + b \lg t$

$\lg D = b(\lg t) + \lg a$

lg *t*	0	0.23	0.30	0.43
lg *D*	0.69	1.15	1.29	1.55



By drawing the graph of lg D against lg t, a straight line is obtained where the gradient, $m = \dfrac{1.55 - 1.15}{0.43 - 0.23} = 2$ and y-intercept, $c = 0.69$

$\lg D = 2(\lg t) + 0.69$

$= \lg t^2 + 0.69$

$\lg D - \lg t^2 = 0.69$

$\lg\left(\dfrac{D}{t^2}\right) = 0.69$

$\dfrac{D}{t^2} = 10^{0.69}$

$D = 10^{0.69}\, t^2$

$\therefore a = 10^{0.69} = 4.9,\ b = 2$

(b) $t = 3,\ \lg 3 = 0.48$

$\lg D = 1.63$

$D = 10^{1.65} = 42.66$ m

(c) $D = 40,\ \lg 40 = 1.6$

$\lg t = 0.455$

$t = 10^{0.455}$

$= 2.85$ seconds