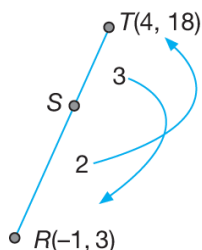


Form 4 Chapter 7
Coordinate Geometry
Fully-Worked Solutions

UPSKILL 7.1

1 (a)

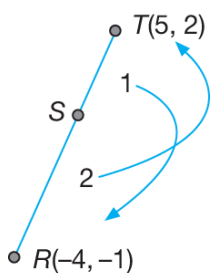


$$S = \left(\frac{3(-1) + 2(4)}{2+3}, \frac{3(3) + 2(18)}{2+3} \right)$$

$$S = \left(\frac{5}{5}, \frac{45}{5} \right)$$

$$S = (1, 9)$$

(b)



$$S = \left(\frac{1(-4) + 2(5)}{2+1}, \frac{1(-1) + 2(2)}{2+1} \right)$$

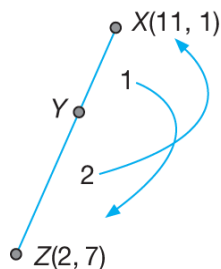
$$S = \left(\frac{6}{3}, \frac{3}{3} \right)$$

$$S = (2, 1)$$

2 (a) $2XY = YZ$

$$\frac{XY}{YZ} = \frac{1}{2}$$

$$XY : YZ = 1 : 2$$



$$Y = \left(\frac{2(11) + 1(2)}{1+2}, \frac{2(1) + 1(7)}{2+1} \right)$$

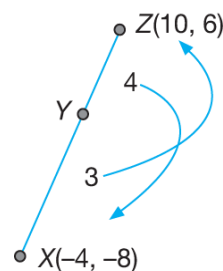
$$Y = \left(\frac{24}{3}, \frac{9}{3} \right)$$

$$Y = (8, 3)$$

(b) $4XY = 3YZ$

$$\frac{XY}{YZ} = \frac{3}{4}$$

$$XY : YZ = 3 : 4$$

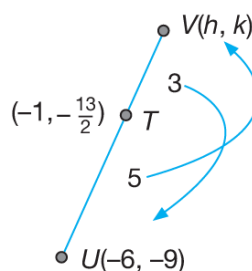


$$Y = \left(\frac{4(-4) + 3(10)}{3+4}, \frac{4(-8) + 3(6)}{3+4} \right)$$

$$Y = \left(\frac{14}{7}, \frac{-14}{7} \right)$$

$$Y = (2, -2)$$

3



Let $V = (h, k)$

$$T = \left(\frac{3(-6) + 5h}{5+3}, \frac{3(-9) + 5k}{5+3} \right)$$

$$T = \left(\frac{5h-18}{8}, \frac{5k-27}{8} \right)$$

But it is given that $T = \left(-1, -\frac{13}{2} \right)$.

Equating the x -coordinates:

$$\frac{5h-18}{8} = -1$$

$$5h-18 = -8$$

$$5h = 10$$

$$h = 2$$

Equating the y -coordinates:

$$\frac{5k-27}{8} = -\frac{13}{2}$$

$$5k-27 = -\frac{13}{2} \times 8$$

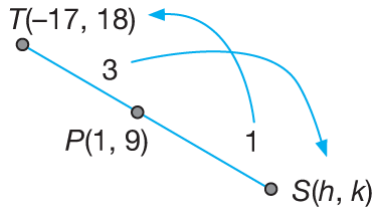
$$5k = -52 + 27$$

$$5k = -25$$

$$k = -5$$

Hence, the coordinates of point V are $(2, -5)$.

4



$$P = \left(\frac{3h + 1(-17)}{1 + 3}, \frac{3k + 1(18)}{1 + 3} \right)$$

$$P = \left(\frac{3h - 17}{4}, \frac{3k + 18}{4} \right)$$

But it is given that the coordinates of point P are $(1, 9)$.

Equating the x -coordinates:

$$\frac{3h - 17}{4} = 1$$

$$3h - 17 = 4$$

$$3h = 21$$

$$h = 7$$

Equating the y -coordinates:

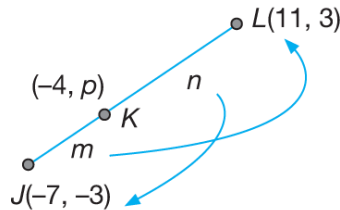
$$\frac{3k + 18}{4} = 9$$

$$3k = 18$$

$$k = 6$$

Hence, the coordinates of point S are $(7, 6)$.

5



(a) Equating the x -coordinates:

$$\frac{n(-7) + m(11)}{m + n} = -4$$

$$-7n + 11m = -4m - 4n$$

$$15m = 3n$$

$$\frac{m}{n} = \frac{3}{15}$$

$$\frac{m}{n} = \frac{1}{5}$$

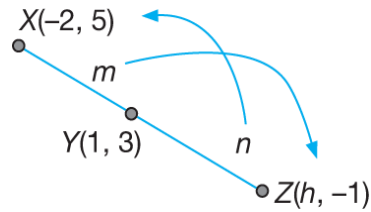
$$m : n = 1 : 5$$

$$JK : KL = 1 : 5$$

(b) Equating the y -coordinates:

$$p = \frac{5(-3) + 1(3)}{1 + 5} = -\frac{12}{6} = -2$$

6



(a) Equating the y -coordinates:

$$\frac{n(5) + m(-1)}{m + n} = 3$$

$$5n - m = 3m + 3n$$

$$4m = 2n$$

$$\frac{m}{n} = \frac{1}{2}$$

$$m : n = 1 : 2$$

$$XY : YZ = 1 : 2$$

(b) Equating the x -coordinates:

$$\frac{2(-2) + 1(h)}{1 + 2} = 1$$

$$-4 + h = 3$$

$$h = 7$$

UPSKILL 7.2a

1 (a) $y - y_1 = m(x - x_1)$

$$y - 2 = -2(x - 5)$$

$$y - 2 = -2x + 10$$

$$y = -2x + 12$$

(b) $y - 3 = \frac{3}{4}(x + 8)$

$$4(y - 3) = 3(x + 8)$$

$$4y - 12 = 3x + 24$$

$$4y = 3x + 36$$

2 (a) $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$

$$\frac{y + 1}{x - 2} = \frac{0 + 1}{3 - 2}$$

$$\frac{y + 1}{x - 2} = 1$$

$$y + 1 = x - 2$$

$$y = x - 2 - 1$$

$$y = x - 3$$

$$\begin{aligned}
 \text{(b)} \quad \frac{y-3}{x+4} &= \frac{-5-3}{2+4} \\
 \frac{y-3}{x+4} &= \frac{-5-3}{2+4} \\
 \frac{y-3}{x+4} &= \frac{-4}{3} \\
 3(y-3) &= -4(x+4) \\
 3y-9 &= -4x-16 \\
 3y &= -4x-7
 \end{aligned}$$

$$3 \text{ (a)} \quad \frac{x}{3} + \frac{y}{(-5)} = 1$$

$$\text{(b)} \quad \frac{x}{(-8)} + \frac{y}{(-6)} = 1$$

$$\text{(c)} \quad \frac{x}{\left(-\frac{1}{2}\right)} + \frac{y}{\left(\frac{3}{4}\right)} = 1$$

$$\begin{aligned}
 \text{4 (a)} \quad \frac{y-3}{x+1} &= \frac{15-3}{5+1} \\
 \frac{y-3}{x+1} &= 2 \\
 y-3 &= 2x+2 \\
 y &= 2x+5
 \end{aligned}$$

$$\text{(b)} \quad 2x - y + 5 = 0$$

$$\begin{aligned}
 \text{(c)} \quad -\frac{2x}{5} + \frac{y}{5} &= \frac{5}{5} \\
 -\frac{x}{5} + \frac{y}{5} &= 1 \\
 -\frac{x}{5} + \frac{y}{5} &= 1
 \end{aligned}$$

$$x\text{-intercept} = -\frac{5}{2}$$

$$y\text{-intercept} = 5$$

$$\text{Gradient} = 2$$

5 The equation of PQ is

$$y - 8 = 3(x - 2)$$

$$y - 8 = 3x - 6$$

$$y = 3x + 2 \dots (1)$$

The equation of RS is

$$\frac{y+2}{x+6} = \frac{6+2}{2+6}$$

$$\frac{y+2}{x+6} = 1$$

$$y+2 = x+6$$

$$y = x + 4 \dots (2)$$

$$y = 3x + 2 \dots (1)$$

$$y = x + 4 \dots (2)$$

Substitute (2) into (1) :

$$x + 4 = 3x + 2$$

$$2x = 2$$

$$x = 1$$

From (2) :

$$y = 1 + 4$$

$$y = 5$$

Hence, the coordinates of the point of intersection are (1, 5).

6 Point P

$$\text{Equation of } PQ: 3y = x + 7 \dots (1)$$

$$\text{Equation of } PR: 7y = -3x - 5 \dots (2)$$

$$(1) \times 3: 9y = 3x + 21 \dots (3)$$

$$(2) + (3): 16y = 16$$

$$y = 1$$

$$\text{From (1): } 3(1) = x + 7$$

$$x = -4$$

The coordinates of point P are (-4, 1).

Point Q

$$\text{Equation of } PQ: 3y = x + 7 \dots (1)$$

$$\text{Equation of } QR: y = -5x + 13 \dots (2)$$

$$(2) \times 3: 3y = -15x + 39 \dots (3)$$

$$(1) - (3): 16x - 32 = 0$$

$$x = 2$$

$$\text{From (2): } y = -5(2) + 13 = 3$$

The coordinates of point Q are (2, 3).

Point R

$$\text{Equation of } PR: 7y = -3x - 5 \dots (1)$$

$$\text{Equation of } QR: y = -5x + 13 \dots (2)$$

$$(2) \times 7: 7y = -35x + 91 \dots (3)$$

$$(1) - (3): 0 = 32x - 96$$

$$x = 3$$

$$\text{From (2): } y = -5(3) + 13 = -2$$

The coordinates of point R are (3, -2).

7 The equation of the straight line CD is

$$\frac{y-2}{x-4} = \frac{-4-2}{-5-4}$$

$$\frac{y-2}{x-4} = \frac{2}{3}$$

$$3y - 6 = 2x - 8$$

$$3y = 2x - 2$$

$$\text{Equation of } CD: 3y = 2x - 2 \dots (1)$$

$$\text{Equation of } ST: 2y = 7x + 10 \dots (2)$$

$$(1) \times 2 : 6y = 4x - 4 \quad \dots (3)$$

$$(2) \times 3 : 6y = 21x + 30 \quad \dots (4)$$

$$(3) - (4) : 0 = -17x - 34$$

$$x = -2$$

$$\text{From (1) : } 3y = 2(-2) - 2$$

$$y = -2$$

The coordinates of point P are $(-2, -2)$.

The equation of the straight line HK is

$$y + 2 = -2(x + 2)$$

$$y = -2x - 6$$

UPSKILL 7.2b

$$1 \text{ Gradient } PQ = \frac{8-18}{5-1} = -\frac{5}{2}$$

$$\text{Gradient } TU = \frac{8+2}{-5+1} = -\frac{5}{2}$$

Hence, PQ is parallel to TU .

$$2 \text{ } 3x + 2y - 1 = 0$$

$$2y = -3x + 1$$

$$y = -\frac{3}{2}x + \frac{1}{2}$$

$$m_1 = -\frac{3}{2}$$

$$-\frac{x}{2} - \frac{y}{3} = 1$$

$$m_2 = -\frac{y\text{-intercept}}{x\text{-intercept}}$$

$$= -\frac{-3}{-2}$$

$$= -\frac{3}{2}$$

Hence, both straight lines are parallel.

$$3 \text{ } 5x - 2y + 4 = 0$$

$$-2y = -5x - 4$$

$$y = \frac{5}{2}x + 2$$

$$m = \frac{5}{2}$$

$$y - 4 = \frac{5}{2}(x - 5)$$

$$2y - 8 = 5x - 25$$

$$2y = 5x - 17$$

UPSKILL 7.2c

$$1 \text{ } m_1 = \text{Gradient of } PQ = \frac{4-3}{8-4} = \frac{1}{4}$$

$$m_2 = \text{Gradient of } TU = \frac{5-1}{6-7} = -4$$

$$m_1 m_2 = \frac{1}{4} \times (-4) = -1$$

Hence, PQ is perpendicular to TU .

$$2 \text{ } m_1 = \text{Gradient of } KL = \frac{5-4}{9-1} = \frac{1}{8}$$

$$2x + \frac{y}{4} = 3$$

$$8x + y = 12$$

$$y = -8x + 12$$

$$m_2 = \text{Gradient of } MN = -8$$

$$m_1 m_2 = \frac{1}{8} \times (-8) = -1$$

Hence, MN is perpendicular to TU .

$$3 \text{ } 4x - 3y + 8 = 0$$

$$3y = 4x + 8$$

$$y = \frac{4}{3}x + \frac{8}{3}$$

$$m_1 = \frac{4}{3}$$

$$\frac{x}{8} + \frac{y}{6} = 1$$

$$m_2 = -\frac{6}{8} = -\frac{3}{4}$$

$$m_1 m_2 = \frac{4}{3} \times \left(-\frac{3}{4}\right) = -1$$

Hence, the straight line $4x - 3y + 8 = 0$ is

perpendicular to the straight line $\frac{x}{8} + \frac{y}{6} = 1$.

$$4 \text{ } 4x + 3y + 2 = 0$$

$$3y = -4x - 2$$

$$y = -\frac{4}{3}x - \frac{2}{3}$$

$$m = -\frac{4}{3}$$

Gradient of the perpendicular line

$$= \frac{3}{4}$$

The equation of the perpendicular line is

$$y - 3 = \frac{3}{4}(x + 2)$$

$$4y - 12 = 3x + 6$$

$$4y = 3x + 18$$

$$5 \quad N = \text{Midpoint} = \left(\frac{3-3}{2}, \frac{-1+6}{2} \right) = \left(0, \frac{5}{2} \right)$$

$$m_{PS} = \frac{-4-2}{1+5} = -1$$

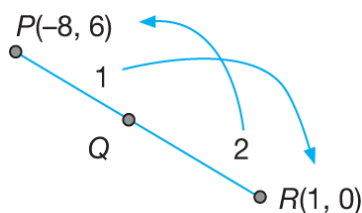
Thus, the gradient the perpendicular line is 1.
The equation of the perpendicular line is

$$y - \frac{5}{2} = (1)(x - 0)$$

$$2y - 5 = 2x$$

$$2y = 2x + 5$$

6 (a)



$$Q = \left(\frac{2(-8)+1(1)}{1+2}, \frac{1(0)+2(6)}{1+2} \right)$$

$$Q = (-5, 4)$$

$$m_{PQR} = \frac{0-6}{1+8} = -\frac{2}{3}$$

Gradient of the perpendicular line is $\frac{3}{2}$.

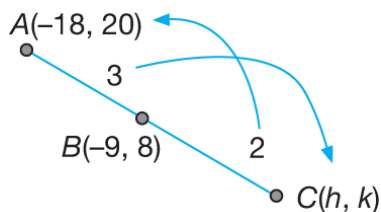
The equation of the perpendicular line is

$$y - 4 = \frac{3}{2}(x + 5)$$

$$2y - 8 = 3x + 15$$

$$2y = 3x + 23$$

(b) Let C be point (h, k) .



$$\left(\frac{2(-18)+3h}{3+2}, \frac{2(20)+3k}{3+2} \right) = (-9, 8)$$

$$\left(\frac{3h-36}{5}, \frac{40+3k}{5} \right) = (-9, 8)$$

Equating the x -coordinates,

$$\frac{3h-36}{5} = -9$$

$$3h - 36 = -45$$

$$3h = -9$$

$$h = -3$$

Equating the y -coordinates,

$$\frac{40+3k}{5} = 8$$

$$40+3k = 40$$

$$3k = 0$$

$$k = 0$$

Hence, the coordinates of point C are $(-3, 0)$.

$$m_{ABC} = \frac{0-20}{-3+18} = -\frac{4}{3}$$

Gradient of the perpendicular line is

$$\frac{3}{4}$$

The equation of perpendicular line is

$$y - 0 = \frac{3}{4}(x + 3)$$

$$4y = 3(x + 3)$$

$$4y = 3x + 9$$

7 Midpoint of KL is

$$\left(\frac{-2+3}{2}, \frac{3+6}{2} \right) = \left(\frac{1}{2}, \frac{9}{2} \right)$$

$$m_{KL} = \frac{6-3}{3+2} = \frac{3}{5}$$

Gradient of perpendicular line = $-\frac{5}{3}$

The equation of the perpendicular bisector is

$$y - \frac{9}{2} = -\frac{5}{3} \left(x - \frac{1}{2} \right)$$

$$6y - 27 = -10 \left(x - \frac{1}{2} \right)$$

$$6y - 27 = -10x + 5$$

$$6y = -10x + 32$$

$$3y = -5x + 16$$

8 (a) Midpoint of BC

$$= \left(\frac{5-1}{2}, \frac{3+5}{2} \right)$$

$$= (2, 4)$$

$$m_{BC} = \frac{5-3}{-1-5} = -\frac{1}{3}$$

Gradient of the perpendicular line = 3

The equation of the perpendicular bisector is

$$y - 4 = 3(x - 2)$$

$$y - 4 = 3x - 6$$

$$y = 3x - 2 \dots (1)$$

(b) The equation of the straight line AB is

$$\frac{y+4}{x+2} = \frac{3+4}{5+2}$$

$$\frac{y+4}{x+2} = 1$$

$$y+4 = x+2$$

$$y = x - 2 \quad \dots (2)$$

Substitute (1) into (2) :

$$3x - 2 = x - 2$$

$$2x = 0$$

$$x = 0$$

From (2) :

$$y = x - 2 = 0 - 2 = -2$$

Hence, the required point of intersection is $(0, -2)$.

9 (a) $x + y = 8$

$$y = -x + 8$$

$$m_1 = -1$$

$$m_{PR} = 1$$

The equation of PR is

$$y - 1 = 1(x - 2)$$

$$y = x - 1$$

(b) $m_{SQ} = -1$

The equation of SQ is

$$y - 2 = -(x - 5)$$

$$y = -x + 5 + 2$$

$$y = -x + 7$$

(c) Equation of PTR : $y = x - 1 \quad \dots (1)$

Equation of STQ : $y = -x + 7 \quad \dots (2)$

Substitute (1) into (2) :

$$x - 1 = -x + 7$$

$$2x = 8$$

$$x = 4$$

Substitute $x = 4$ into (1) :

$$y = 4 - 1 = 3$$

Hence, T is point $(4, 3)$.

Let R be point (h, k) .

T is the midpoint of PR .

$$\frac{2+h}{2} = 4 \quad \frac{1+k}{2} = 3$$

$$h = 6 \quad k = 5$$

Hence, R is point $(6, 5)$.

Let S be point (a, b) .

T is the midpoint of SQ .

$$\frac{5+a}{2} = 4 \quad \frac{2+b}{2} = 3$$

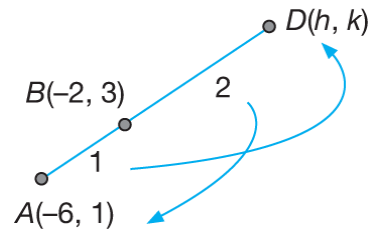
$$a = 3 \quad b = 4$$

Hence, S is point $(3, 4)$.

10 (a) $AB = \frac{1}{2}BD$

$$\frac{AB}{BD} = \frac{1}{2}$$

$$AB : BD = 1 : 2$$



$$\left(\frac{2(-6)+h}{1+2}, \frac{2(1)+k}{1+2} \right) = (-2, 3)$$

$$\left(\frac{h-12}{3}, \frac{k+2}{3} \right) = (-2, 3)$$

Equating the x -coordinates:

$$\frac{h-12}{3} = -2$$

$$h = 6$$

Equating the y -coordinates:

$$\frac{k+2}{3} = 3$$

$$k + 2 = 9$$

$$k = 7$$

Hence, D is point $(6, 7)$.

(b) $m_{ABD} = \frac{3-1}{-2+6} = \frac{1}{2}$

Thus, $m_{CE} = -2$

Hence, the equation of CE is

$$y - 0 = -2(x - 7)$$

$$y = -2x + 14 \quad \dots (1)$$

(c) The equation of $ABCD$:

$$y - 1 = \frac{1}{2}(x + 6)$$

$$2y - 2 = x + 6$$

$$2y = x + 8 \quad \dots (2)$$

Substitute (1) into (2) :

$$2(-2x + 14) = x + 8$$

$$-4x + 28 = x + 8$$

$$5x = 20$$

$$x = 4$$

Substitute $x = 4$ into (1) :

$$y = -2(4) + 14$$

$$y = 6$$

Hence, C is point $(4, 6)$.

UPSKILL 7.3

1 (a) Area of triangle PQR

$$\begin{aligned}
 &= \frac{1}{2} \begin{vmatrix} -4 & 1 & 3 & -4 \\ 1 & 2 & 5 & 1 \end{vmatrix} \\
 &= \frac{1}{2} |-8 + 5 + 3 - (1 + 6 - 20)| \\
 &= \frac{1}{2} |0 - (-13)| \\
 &= \frac{13}{2} \text{ units}^2
 \end{aligned}$$

(b) Area of triangle KLM

$$\begin{aligned}
 &= \frac{1}{2} \begin{vmatrix} 2 & 9 & 5 & 2 \\ 3 & 4 & 1 & 3 \end{vmatrix} \\
 &= \frac{1}{2} |8 + 9 + 15 - (27 + 20 + 2)| \\
 &= \frac{1}{2} |32 - 49| \\
 &= \frac{1}{2} |-17| \\
 &= \frac{17}{2} \text{ units}^2
 \end{aligned}$$

(c) Area of triangle ABC

$$\begin{aligned}
 &= \frac{1}{2} \begin{vmatrix} -2 & 4 & 1 & -2 \\ 1 & 2 & -3 & 1 \end{vmatrix} \\
 &= \frac{1}{2} |-4 - 12 + 1 - (4 + 2 + 6)| \\
 &= \frac{1}{2} |-15 - 12| \\
 &= \frac{1}{2} |-27| \\
 &= \frac{27}{2} \text{ units}^2
 \end{aligned}$$

2 (a) Area of the quadrilateral $EFGH$

$$\begin{aligned}
 &= \frac{1}{2} \begin{vmatrix} 2 & -3 & -2 & 4 & 2 \\ -2 & -1 & 5 & 0 & -2 \end{vmatrix} \\
 &= \frac{1}{2} |-2 - 15 - 8 - (6 + 2 + 20)| \\
 &= \frac{1}{2} |-25 - 28|
 \end{aligned}$$

$$= \frac{1}{2} |-53|$$

$$= \frac{53}{2} \text{ units}^2$$

(b) Area of the quadrilateral $ABCD$

$$\begin{aligned}
 &= \frac{1}{2} \begin{vmatrix} -5 & 0 & 6 & 0 & -5 \\ 0 & 10 & 4 & 0 & 0 \end{vmatrix} \\
 &= \frac{1}{2} |-50 - (60)| \\
 &= \frac{1}{2} |-110| \\
 &= \frac{1}{2} (110) \\
 &= 55 \text{ units}^2
 \end{aligned}$$

3 Area of $\triangle ABC = 12 \text{ units}^2$

$$\frac{1}{2} \begin{vmatrix} 1 & 7 & h & 1 \\ 2 & 8 & -10 & 2 \end{vmatrix} = 12$$

$$\begin{vmatrix} 1 & 7 & h & 1 \\ 2 & 8 & -10 & 2 \end{vmatrix} = 24$$

$$|8 - 70 + 2h - (14 + 8h - 10)| = 24$$

$$|-66 - 6h| = 24$$

$$|-11 - h| = 4$$

$$-11 - h = \pm 4$$

$$-11 - h = 4 \quad \text{or} \quad -11 - h = -4$$

$$-h = 15$$

$$h = -7$$

$$h = -15$$

4 Area of the quadrilateral $TUVW = 18 \text{ units}^2$

$$\frac{1}{2} \begin{vmatrix} -1 & 3 & 4 & -1 \\ 3 & 1 & 5 & 3 \end{vmatrix} = 18$$

$$|1 + p + 15 + 12 - (3p - 3 + 4 - 5)| = 18$$

$$|32 - 2p| = 36$$

$$32 - 2p = \pm 36$$

$$32 - 2p = 36 \quad \text{or} \quad 32 - 2p = -36$$

$$-2p = 4$$

$$-2p = -68$$

$$p = -2$$

$$p = 34$$

5 Area of $\triangle LMN = 0$

$$\frac{1}{2} \begin{vmatrix} -2 & 1 & 4 & -2 \\ 4 & h & 4 & 4 \end{vmatrix} = 0$$

$$-2h + 4h + 16 - (4 + 4h - 8h) = 0$$

$$6h + 12 = 0$$

$$h = -2$$

6 (a)(i) Area of $\triangle ABC$

$$= \frac{1}{2} \left| \begin{array}{ccc} -8 & 6 & -1 \\ -4 & -6 & 2 \\ -8 & -4 & -4 \end{array} \right|$$

$$= \frac{1}{2} |48 + 12 + 4 - (-24 + 6 - 16)|$$

$$= \frac{1}{2} |98|$$

$$= 49 \text{ units}^2$$

(ii) $AC = \sqrt{(6+8)^2 + (-6+4)^2}$

$$AC = \sqrt{196+4}$$

$$AC = \sqrt{200}$$

$$AC = 14.1421 \text{ units}$$

(b) Let the perpendicular distance from point B to the straight line AC be h units.

$$\text{Area of } \triangle ABC = 49$$

$$\frac{1}{2} \times \text{Base} \times \text{Height} = 49$$

$$\frac{1}{2} \times 14.1421 \times h = 49$$

$$h = 6.930$$

Hence, the shortest distance from B to AC is 6.930 units.

UPSKILL 7.4

1 $PA = 5$

$$\sqrt{(x-4)^2 + (y-6)^2} = 5$$

$$(x-4)^2 + (y-6)^2 = 5^2$$

$$x^2 - 8x + 16 + y^2 - 12y + 36 - 25 = 0$$

$$x^2 - 8x + y^2 - 12y + 27 = 0$$

2 $BQ = CQ$
 $BQ^2 = CQ^2$

$$(x-5)^2 + (y-1)^2 = (x+4)^2 + (y-2)^2$$

$$x^2 - 10x + 25 + y^2 - 2y + 1 =$$

$$x^2 + 8x + 16 + y^2 - 4y + 4$$

$$-18x + 2y + 6 = 0$$

$$-9x + y + 3 = 0$$

$$y = 9x - 3$$

3 $\frac{HJ}{HK} = \frac{2}{1}$

$$HJ = 2HK$$

$$HJ^2 = 2^2 HK^2$$

$$(x-3)^2 + (y-0)^2 = 4[(x-0)^2 + (y+2)^2]$$

$$x^2 - 6x + 9 + y^2 = 4(x^2 + y^2 + 4y + 4)$$

$$x^2 - 6x + 9 + y^2 = 4x^2 + 4y^2 + 16y + 16$$

$$3x^2 + 3y^2 + 6x + 16y + 7 = 0$$

4 $\frac{KP}{KQ} = \frac{3}{2}$

$$2KP = 3KQ$$

$$4KP^2 = 9KQ^2$$

$$4[(x-2)^2 + (y-1)^2] = 9[(x+1)^2 + (y+2)^2]$$

$$4(x^2 - 4x + 4 + y^2 - 2y + 1) =$$

$$9(x^2 + 2x + 1 + y^2 + 4y + 4)$$

$$4x^2 - 16x + 4y^2 - 8y + 20 =$$

$$9x^2 + 18x + 9y^2 + 36y + 45$$

$$5x^2 + 5y^2 + 34x + 44y + 25 = 0$$

5 $JM = 2JN$

$$JM^2 = 2^2 JN^2$$

$$(x-0)^2 + (y+1)^2 = 4[(x-2)^2 + (y-0)^2]$$

$$x^2 + y^2 + 2y + 1 = 4(x^2 - 4x + 4 + y^2)$$

$$x^2 + y^2 + 2y + 1 = 4x^2 - 16x + 16 + 4y^2$$

$$3x^2 + 3y^2 - 16x - 2y + 15 = 0$$

(a) At the y -axis, $x = 0$

$$3y^2 - 2y + 15 = 0$$

$$b^2 - 4ac = (-2)^2 - 4(3)(15) = -176$$

Since $b^2 - 4ac < 0$, the locus of J does not intersect the y -axis.

(b) At the x -axis, $y = 0$

$$3x^2 - 16x + 15 = 0$$

$$b^2 - 4ac = (-16)^2 - 4(3)(15) = 76$$

Since $b^2 - 4ac > 0$, the locus of J will intersect the x -axis.

6 Since $\angle APB = 90^\circ$

$$m_{AP} \times m_{PB} = -1$$

$$\left(\frac{y+2}{x-3}\right)\left(\frac{y-0}{x-6}\right) = -1$$

$$\frac{y^2 + 2y}{x^2 - 9x + 18} = -1$$

$$y^2 + 2y = -x^2 + 9x - 18$$

$$x^2 + y^2 - 9x + 2y + 18 = 0$$

7 Since $\angle APB = 90^\circ$

$$m_{AP} \times m_{PB} = -1$$

$$\left(\frac{y-2}{x-0}\right)\left(\frac{y+2}{x-2}\right) = -1$$

$$\frac{y^2-4}{x^2-2x} = -1$$

$$y^2-4 = -x^2+2x$$

$$x^2+y^2-2x-4=0$$

8 Since $\angle MQN = 90^\circ$

$$m_{MQ} \times m_{QN} = -1$$

$$\left(\frac{y-4}{x-1}\right)\left(\frac{y-0}{x-3}\right) = -1$$

$$\frac{y^2-4y}{x^2-4x+3} = -1$$

$$y^2-4y = -x^2+4x-3$$

$$x^2+y^2-4x-4y+3=0$$

Summative Practice 7

1 (a) $m_{AB} = m_{DC} = 2$

The equation of AB is

$$y-7 = 2(x-2)$$

$$y-7 = 2x-4$$

$$y = 2x+3$$

(b) $m_{AD} = -\frac{1}{m_{DC}} = -\frac{1}{2}$

The equation of AD is

$$y-(-1) = -\frac{1}{2}(x-3)$$

$$2y+2 = -x+3$$

$$2y = -x+1$$

(c) $y = 2x+3 \dots (1)$
 $2y = -x+1 \dots (2)$

Substitute (1) into (2):

$$2(2x+3) = -x+1$$

$$4x+6 = -x+1$$

$$5x = -5$$

$$x = -1$$

From (1):

$$y = 2(-1)+3 = 1$$

Hence, the coordinates of point A are $(-1, 1)$.

(d) Area of $\triangle BAD$

$$= \frac{1}{2} \begin{vmatrix} 2 & -1 & 3 & 2 \\ 7 & 1 & -1 & 7 \end{vmatrix}$$

$$= \frac{1}{2} |2+1+21-(-7+3-2)|$$

$$= 15 \text{ units}^2$$

Hence, the area of the rectangle $ABCD$

$$= 2 \times 15 = 30 \text{ units}^2$$

2 (a) $m_{PQ} = \frac{9-7}{6-2} = \frac{1}{2}$

$$m_{PS} = -2$$

The equation of PS is

$$y-7 = -2(x-2)$$

$$y-7 = -2x+4$$

$$y = -2x+11$$

(b) Substitute $y = -2x+11$ into

$$7x-2y = 44,$$

$$7x-2(-2x+11) = 44$$

$$7x+4x-22 = 44$$

$$11x = 66$$

$$x = 6$$

When $x = 6$, $y = -2(6)+11 = -1$

Hence, S is point $(6, -1)$.

(c) $T = \left(\frac{3(2)+1(6)}{1+3}, \frac{3(7)+1(-1)}{1+3}\right)$
 $= (3, 5)$

(d) Area of $PQRS = 30 \text{ units}^2$

$$\frac{1}{2} \begin{vmatrix} 2 & 6 & k & 2 \\ 7 & -1 & 7k-44 & 9 \end{vmatrix} = 30$$

When $x = k$,

$$7x-2y = 44$$

$$7k-2y = 44$$

$$y = \frac{7k-44}{2}$$

$$-2+3(7k-44)+9k+42$$

$$-[42-k+3(7k-44)+18] = 60$$

$$-2+3(7k-44)+9k+42$$

$$-42+k-3(7k-44)-18 = 60$$

$$-20+10k = 60$$

$$k = 8$$

When $k = 8$, $y = \frac{7(8)-44}{2} = 6$

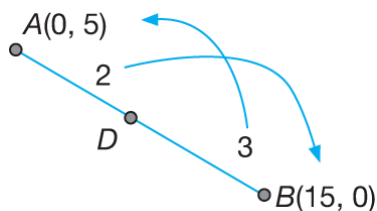
Hence, R is point $(8, 6)$.

3 (a) The equation of AB is $\frac{x}{15} + \frac{y}{5} = 1$.

(b) $3AD = 2DB$

$$\frac{AD}{DB} = \frac{2}{3}$$

$$AD:DB = 2:3$$



$$D = \left(\frac{3(0) + 2(15)}{2+3}, \frac{3(5) + 2(0)}{2+3} \right)$$

$$D = (6, 3)$$

(c) $m_{AB} = \frac{0-5}{15-0} = -\frac{1}{3}$

$$m_{CD} = 3$$

The equation of CD is

$$y - 3 = 3(x - 6)$$

$$y - 3 = 3x - 18$$

$$y = 3x - 15$$

Hence, the y -intercept is -15 .

4 (a) Equation of AB : $y = 6x - 8 \dots (1)$

Equation of AN : $5y = 2x + 16 \dots (2)$

Substitute (1) into (2):

$$5(6x - 8) = 2x + 16$$

$$30x - 40 = 2x + 16$$

$$28x = 56$$

$$x = 2$$

When $x = 2$,

$$y = 6(2) - 8 = 4$$

Hence, the coordinates of point A are $(2, 4)$.

(b) M is the midpoint of AC .

Let C is point (h, k) .

$$\left(\frac{2+h}{2}, \frac{4+k}{2} \right) = (5, 8)$$

Equating the x -coordinates:

$$\frac{2+h}{2} = 5$$

$$h = 8$$

Equating the y -coordinates:

$$\frac{4+k}{2} = 8$$

$$k = 12$$

Hence, the coordinates of point C are $(8, 12)$.

(c) The equation of AN is

$$5y = 2x + 16 \Rightarrow y = \frac{2}{5}x + \frac{16}{5}$$

$$m_{AN} = \frac{2}{5}$$

$$\text{Thus, } m_{CN} = -\frac{5}{2}$$

The equation of CN is

$$y - 12 = -\frac{5}{2}(x - 8)$$

$$2y - 24 = -5x + 40$$

$$2y = -5x + 64$$

(d) $m_{CD} = m_{AB} = 6$

The equation of CD is

$$y - 12 = 6(x - 8)$$

$$y - 12 = 6x - 48$$

$$y = 6x - 36$$

(e) Equation of CD : $y = 6x - 36 \dots (1)$

Equation of AD : $5y = 2x + 16 \dots (2)$

Substitute (1) into (2):

$$5(6x - 36) = 2x + 16$$

$$30x - 180 = 2x + 16$$

$$28x = 196$$

$$x = 7$$

From (1):

$$y = 6x - 36$$

$$y = 6(7) - 36 = 6$$

Hence, the coordinates of point D are $(7, 6)$.

(f) Area of triangle ADC

$$= \frac{1}{2} \left| \begin{array}{ccc} 2 & 7 & 8 \\ 4 & 6 & 12 \\ 4 & 4 & 4 \end{array} \right|$$

$$= \frac{1}{2} |12 + 84 + 32 - (28 + 48 + 24)|$$

$$= \frac{1}{2} (28)$$

$$= 14 \text{ units}^2$$

Hence, the area of the parallelogram $ABCD$

$$= 14 \times 2$$

$$= 28 \text{ units}^2$$

5 (a) $OT = 2OV$
 $OT^2 = 4OV^2$
 $(10-0)^2 + (p-0)^2 = 4[(-5-0)^2 + (-10-0)^2]$
 $100 + p^2 = 4(25+100)$
 $p^2 = 100 + 400 - 100$
 $p^2 = 400$
 $p = 20$

(b) $m_{UT} = m_{VO} = \frac{0+10}{0+5} = 2$
 The equation of UT is
 $y-10 = 2(x-20)$
 $y-10 = 2x-40$
 $y = 2x-30$

(c) $m_{UV} = -\frac{1}{m_{VO}} = -\frac{1}{2}$
 The equation of UV is
 $y+10 = -\frac{1}{2}(x+5)$
 $2y+20 = -x-5$
 $2y = -x-5-20$
 $2y = -x-25$

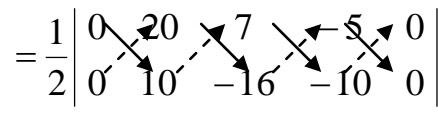
(d) Equation of UT : $y = 2x - 30 \dots (1)$
 Equation of UV : $2y = -x - 25 \dots (2)$

Substitute (1) into (2):
 $2y = -x - 25$
 $2(2x - 30) = -x - 25$
 $4x - 60 = -x - 25$
 $5x = 35$
 $x = 7$

From (1):
 $y = 2x - 30$
 $y = 2(7) - 30$
 $y = -16$
 Hence, U is point $(7, -16)$.

(e) $O(0, 0)$, $T(20, 10)$, $U(7, -16)$, $V(-5, -10)$

Area of $OTUV$



$$= \frac{1}{2} \begin{vmatrix} 0 & 20 & 7 & -5 & 0 \\ 0 & 10 & -16 & -10 & 0 \end{vmatrix}$$

$$= \frac{1}{2} |20(-16) + 7(-10) - (70 + 80)|$$

$$= \frac{1}{2} |-320 - 70 - 150|$$

$$= \frac{1}{2} |-540|$$

$$= 270 \text{ units}^2$$

6 (a) The equation of SQ is
 $x + 2y - 4 = 0$

$$2y = -x + 4$$

$$y = -\frac{1}{2}x + 2$$

$$m_{SQ} = -\frac{1}{2}$$

$$m_{PR} = 2$$

The equation of PR is

$$y - 5 = 2(x + 1)$$

$$y = 2x + 2 + 5$$

$$y = 2x + 7$$

(b) At point Q , (x -axis), $y = 0$:

$$y = -\frac{1}{2}x + 2$$

$$0 = -\frac{1}{2}x + 2$$

$$0 = -x + 4$$

$$x = 4$$

Q is point $(4, 0)$.

Equation of PTR : $y = 2x + 7 \dots (1)$

Equation of STQ : $y = -\frac{1}{2}x + 2 \dots (2)$

Substitute (1) into (2):

$$2x + 7 = -\frac{1}{2}x + 2$$

$$4x + 14 = -x + 4$$

$$5x = -10$$

$$x = -2$$

From (1):

$$y = 2x + 7$$

$$y = 2(-2) + 7$$

$$y = 3$$

Hence, T is point $(-2, 3)$.

Let R is point (h, k) .

$$\left(\frac{h-1}{2}, \frac{k+5}{2}\right) = (-2, 3)$$

$$\frac{h-1}{2} = -2 \qquad \frac{k+5}{2} = 3$$

$$h = -3 \qquad k = 1$$

Hence, R is point $(-3, 1)$.

Let S is point (a, b) .

$$\left(\frac{a+4}{2}, \frac{b+0}{2}\right) = (-2, 3)$$

$$\frac{a+4}{2} = -2 \qquad \frac{b}{2} = 3$$

$$a = -8 \qquad b = 6$$

Hence, S is point $(-8, 6)$.

- (c) $P(-1, 5), Q(4, 0), R(-3, 1), S(-8, 6)$

Area of $PQRS$

$$= \frac{1}{2} \begin{vmatrix} -1 & 4 & -3 & -8 & -1 \\ 5 & 0 & 1 & 6 & 5 \end{vmatrix}$$

$$= \frac{1}{2} |4 - 18 - 40 - (20 - 8 - 6)|$$

$$= \frac{1}{2} |-60|$$

$$= 30 \text{ units}^2$$

- 7 (a) Midpoint of PR

$$= \left(\frac{-1+5}{2}, \frac{5+1}{2} \right)$$

$$= (2, 3)$$

- (b) $5x + y - 2 = 0$

$$y = -5x + 2$$

$$\therefore m_{QS} = -5$$

The equation of QS is

$$y - 3 = -5(x - 2)$$

$$y = -5x + 10 + 3$$

$$y = -5x + 13$$

- (c) $m_{PR} = \frac{1-5}{5-(-1)} = -\frac{2}{3}$

$$m_{QR} = \frac{3}{2}$$

The equation of QR is

$$y - 1 = \frac{3}{2}(x - 5)$$

$$2y - 2 = 3x - 15$$

$$2y = 3x - 13$$

- (d) (i) Equation of QS : $y = -5x + 13$... (1)

Equation of QR : $2y = 3x - 13$... (2)

Substitute (1) into (2):

$$2y = 3x - 13$$

$$2(-5x + 13) = 3x - 13$$

$$-10x + 26 = 3x - 13$$

$$13x = 39$$

$$x = 3$$

From (1):

$$y = -5x + 13$$

$$y = -5(3) + 13 = -2$$

Hence, Q is point $(3, -2)$.

Let S is point (h, k) .

$$\left(\frac{h+3}{2}, \frac{k-2}{2} \right) = (2, 3)$$

$$\frac{h+3}{2} = 2$$

$$h+3 = 4$$

$$h = 1$$

$$\frac{k-2}{2} = 3$$

$$k-2 = 6$$

$$k = 8$$

Hence, S is point $(1, 8)$.

- (ii) $P(-1, 5), Q(3, -2), R(5, 1), S(1, 8)$

Area of the parallelogram $PQRS$

$$= \frac{1}{2} \begin{vmatrix} -1 & 3 & 5 & 1 & -1 \\ 5 & -2 & 1 & 8 & 5 \end{vmatrix}$$

$$= \frac{1}{2} |2 + 3 + 40 + 5 - (15 - 10 + 1 - 8)|$$

$$= \frac{1}{2} |52|$$

$$= 26 \text{ units}^2$$

- 8 (a) $m_{QP} = \frac{2-8}{4-2} = -3$

$$m_{QR} = \frac{1}{3}$$

The equation of QR is

$$y - 8 = \frac{1}{3}(x - 2)$$

$$3y - 24 = x - 2$$

$$3y = x + 22$$

- (b) Equation of QR : $3y = x + 22$... (1)

Equation of PR : $y = x - 2$... (2)

Substitute (2) into (1):

$$3y = x + 22$$

$$3(x - 2) = x + 22$$

$$3x - 6 = x + 22$$

$$2x = 28$$

$$x = 14$$

From (2):

$$y = x - 2$$

$$y = 14 - 2 = 12$$

Hence, R is point $(14, 12)$.

- (c) Let S is point (h, k) .

Midpoint of QS = Midpoint of PR

$$\left(\frac{h+2}{2}, \frac{k+8}{2} \right) = \left(\frac{4+14}{2}, \frac{2+12}{2} \right)$$

Equating the x -coordinates,

$$h + 2 = 18$$

$$h = 16$$

Equating the y -coordinates,

$$k + 8 = 14$$

$$k = 6$$

Hence, S is point $(16, 6)$.

(d) $P(4, 2), Q(2, 8), R(14, 12), S(16, 6)$

Area of $PQRS$

$$\begin{aligned}
 &= \frac{1}{2} \left| \begin{array}{cccc} 4 & 2 & 14 & 16 \\ 2 & 8 & 12 & 6 \\ 4 & 2 & 14 & 16 \\ 2 & 8 & 12 & 6 \end{array} \right| \\
 &= \frac{1}{2} |32 + 24 + 84 + 32 - (4 + 112 + 192 + 24)| \\
 &= \frac{1}{2} |172 - 332| \\
 &= \frac{1}{2} |-160| \\
 &= 80 \text{ units}^2
 \end{aligned}$$

9 (a) Equation of PQ : $3y = x + 12 \dots (1)$

Equation of RQ : $3y = 5x - 12 \dots (2)$

Substitute (2) into (1):

$$5x - 12 = x + 12$$

$$4x = 24$$

$$x = 6$$

From (1): $3y = 6 + 12$

$$y = 6$$

Hence, Q is point $(6, 6)$.

(b) $m_{OR} = m_{PQ} = \frac{1}{3}$

The equation of OR is

$$y = \frac{1}{3}x \dots (1)$$

The equation of QR is $3y = 5x - 12 \dots (2)$

Substitute (1) into (2):

$$3\left(\frac{1}{3}x\right) = 5x - 12$$

$$x = 5x - 12$$

$$4x = 12$$

$$x = 3$$

From (1):

$$y = \frac{1}{3}(3) = 1$$

Hence, R is point $(3, 1)$.

(c) $m_{PQ} = \frac{1}{3}$

Thus, the gradient of the perpendicular line is -3 .

The equation of the straight line that passes through the point R and is perpendicular to PQ is

$$y - 1 = -3(x - 3)$$

$$y = -3x + 10$$

10 (a) $(m_{BC})(m_{BA}) = -1$

The straight line CB is perpendicular to the straight line BA .

$$\left(\frac{k-8}{h-(-6)}\right)\left(\frac{8-4}{-6-0}\right) = -1$$

$$\left(\frac{k-8}{h+6}\right)\left(\frac{2}{-3}\right) = -1$$

$$2(k-8) = 3(h+6)$$

$$2k - 16 = 3h + 18$$

$$3h - 2k = -34$$

(b) Area of triangle ABC

$$= \frac{1}{2} \left| \begin{array}{ccc} 0 & -6 & h \\ 4 & 8 & k \\ 4 & 4 & 4 \end{array} \right|$$

$$= \frac{1}{2} [-6k + 4h - (-24 + 8h)]$$

$$= \frac{1}{2} (-6k - 4h + 24)$$

$$= -3k - 2h + 12 \text{ [Shown]}$$

(c) If the area of the rectangle $ABCD$ is 104 units², hence the area of $\triangle ABC = 52$ units².

$$-3k - 2h + 12 = 52$$

$$-2h - 3k = 40 \dots (1)$$

From (a):

$$3h - 2k = -34 \dots (2)$$

$$(1) \times 2: -4h - 6k = 80 \dots (3)$$

$$(2) \times 3: 9h - 6k = -102 \dots (4)$$

$$(4) - (3): 13h = -182$$

$$h = -14$$

Substitute $h = -14$ into (1):

$$-2(-14) - 3k = 40$$

$$-3k = 40 - 28$$

$$k = -4$$

Hence, the coordinates point C are

$(-14, -4)$.

(d) $m_{AD} = m_{BC}$

$$m_{AD} = \frac{-4-8}{-14-(-6)} = \frac{3}{2}$$

The equation of AD is

$$y = mx + c$$

$$y = \frac{3}{2}x + 4$$

$$2y = 3x + 8$$

$$3x - 2y = -8$$

$$\frac{3x}{-8} - \frac{2y}{-8} = \frac{-8}{-8}$$

$$\frac{x}{\left(-\frac{8}{3}\right)} + \frac{y}{4} = 1$$

11 (a) Let R is point (x, y) .

$$AR = BR$$

$$AR^2 = BR^2$$

$$[x - (-1)]^2 + (y - 4)^2 = (x - 1)^2 + [y - (-2)]^2$$

$$(x + 1)^2 + (y - 4)^2 = (x - 1)^2 + (y + 2)^2$$

$$x^2 + 2x + 1 + y^2 - 8y + 16 = x^2 - 2x + 1 + y^2 + 4y + 4$$

$$4x - 12y + 12 = 0$$

$$x - 3y + 3 = 0$$

Hence, the equation of PQ

is $x - 3y + 3 = 0$.

(b) (i) $x - 3y + 3 = 0 \dots (1)$

$x + 2y - 7 = 0 \dots (2)$

$(1) - (2) : -5y + 10 = 0$

$y = 2$

From (1) : $x - 3(2) + 3 = 0$

$x = 3$

Hence, the coordinates of the traffic light are $(3, 2)$.

(ii) When $C\left(1, \frac{4}{3}\right)$ is substituted into

$x - 3y + 3 = 0$, then

$$1 - 3\left(\frac{4}{3}\right) + 3$$

$= 0$

When $C\left(1, \frac{4}{3}\right)$ is substituted into

$x + 2y - 7 = 0$, then

$$1 + 2\left(\frac{4}{3}\right) - 7$$

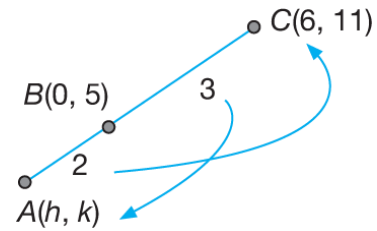
$$= -\frac{10}{3} (\neq 0)$$

Hence, the road PQ passes through

$C\left(1, \frac{4}{3}\right)$ and the road ST does not

pass through the point $C\left(1, \frac{4}{3}\right)$.

12 (a) (i)



Let A be point (h, k) .

$$\left(\frac{3h + 2(6)}{2 + 3}, \frac{3k + 2(11)}{2 + 3}\right) = (0, 5)$$

$$\left(\frac{3h + 12}{5}, \frac{3k + 22}{5}\right) = (0, 5)$$

Equating the x -coordinates,

$$\frac{3h + 12}{5} = 0$$

$$3h + 12 = 0$$

$$3h = -12$$

$$h = -4$$

Equating the y -coordinates,

$$\frac{3k + 22}{5} = 5$$

$$3h + 22 = 25$$

$$3h = 3$$

$$h = 1$$

Hence, the coordinates of point A are $(-4, 1)$.

(ii) The equation of the straight line AD is

$$\frac{y - 1}{x - (-4)} = \frac{-7 - 1}{2 - (-4)}$$

$$\frac{y - 1}{x + 4} = -\frac{4}{3}$$

$$3y - 3 = -4x - 16$$

$$3y = -4x - 13$$

(iii) Area of $\triangle ACD$

$$= \frac{1}{2} \begin{vmatrix} -4 & 6 & 2 & -4 \\ 1 & 1 & -7 & 1 \end{vmatrix}$$

$$= \frac{1}{2} | -44 - 42 + 2 - (6 + 22 + 28) |$$

$$= \frac{1}{2} | -140 |$$

$$= 70 \text{ units}^2$$

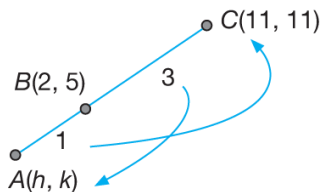
(b) Let P be point (x, y) .

$$\begin{aligned}
 PC &= 2PD \\
 \sqrt{(x-6)^2 + (y-11)^2} &= \\
 2\sqrt{(x-2)^2 + [y-(-7)]^2} \\
 (x-6)^2 + (y-11)^2 &= \\
 2^2[(x-2)^2 + [y-(-7)]^2] \\
 x^2 - 12x + 36 + y^2 - 22y + 121 &= \\
 4[x^2 - 4x + 4 + y^2 + 14y + 49] \\
 x^2 - 12x + y^2 - 22y + 157 &= \\
 4x^2 - 16x + 4y^2 + 56y + 212 \\
 3x^2 + 3y^2 - 4x + 78y + 55 &= 0
 \end{aligned}$$

13 (a) (i) Area of $PQRS$

$$\begin{aligned}
 &= \frac{1}{2} \begin{vmatrix} -5 & 14 & 15 & 0 & -5 \\ 4 & 19 & 2 & 1 & 4 \end{vmatrix} \\
 &= \frac{1}{2} |-95 + 28 + 15 - (56 + 285 - 5)| \\
 &= \frac{1}{2} |-388| \\
 &= 194 \text{ units}^2
 \end{aligned}$$

(ii)



Let A be point (h, k)

$CB : BA = 3 : 1$

$$\left(\frac{1(11) + 3h}{3+1}, \frac{1(11) + 3k}{3+1} \right) = (2, 5)$$

$$\left(\frac{11 + 3h}{4}, \frac{11 + 3k}{4} \right) = (2, 5)$$

Equating the x -coordinates,

$$\frac{11 + 3h}{4} = 2$$

$$11 + 3h = 8$$

$$3h = -3$$

$$h = -1$$

Equating the y -coordinates,

$$\frac{11 + 3k}{4} = 5$$

$$11 + 3k = 20$$

$$3k = 9$$

$$k = 3$$

Hence, A is point $(-1, 3)$.

(b) Let T be point (x, y) .

$$\begin{aligned}
 TC &= 3 \\
 \sqrt{(x-11)^2 + (y-11)^2} &= 3 \\
 (x-11)^2 + (y-11)^2 &= 9 \\
 x^2 - 22x + 121 + y^2 - 22y + 121 &= 9 \\
 x^2 + y^2 - 22x - 22y + 233 &= 0
 \end{aligned}$$

14 (a) (i) The equation of PS is

$$2y = 5x - 23$$

$$y = \frac{5}{2}x - \frac{23}{2}$$

$$m_{PS} = \frac{5}{2}$$

$$m_{PQ} = -\frac{1}{m_{PS}} = -\frac{1}{\left(\frac{5}{2}\right)} = -\frac{2}{5}$$

The equation of PQ is

$$y - (-2) = -\frac{2}{5}[x - (-2)]$$

$$5(y + 2) = -2(x + 2)$$

$$5y + 10 = -2x - 4$$

$$5y = -2x - 14$$

(ii) Equation of PS : $2y = 5x - 23 \dots (1)$

Equation of PQ : $5y = -2x - 14 \dots (2)$

$$10y = 25x - 115 \dots (1) \times 5$$

$$(-) 10y = -4x - 28 \dots (2) \times 2$$

$$0 = 29x - 87$$

$$x = 3$$

When $x = 3$,

$$2y = 5(3) - 23$$

$$y = -4$$

Hence, P is point $(3, -4)$.

(b) The equation of PS is $2y = 5x - 23$.

When $y = 1$, $2(1) = 5x - 23$

$$5x = 25$$

$$x = 5$$

Hence, S is point $(5, 1)$.

Let T be point (x, y) .

$$TS = 5$$

$$\sqrt{(x-5)^2 + (y-1)^2} = 5$$

$$(x-5)^2 + (y-1)^2 = 5^2$$

$$x^2 - 10x + 25 + y^2 - 2y + 1 = 25$$

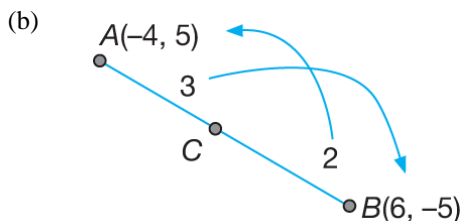
$$x^2 - 10x + y^2 - 2y + 1 = 0$$

15 (a) Area of $\triangle AOB$

$$= \frac{1}{2} \left| \begin{array}{ccc} -4 & 0 & 6 \\ 5 & 0 & -5 \\ -4 & -4 & 5 \end{array} \right|$$

$$= \frac{1}{2} |30 - 20|$$

$$= 5 \text{ units}^2$$



$$C = \left(\frac{2(-4) + 3(6)}{3+2}, \frac{2(5) + 3(-5)}{3+2} \right)$$

$$= (2, -1)$$

(c) Let Q be point (x, y) .

$$QB = 2QA$$

$$\sqrt{(x-6)^2 + [y-(-5)]^2} =$$

$$2\sqrt{[x-(-4)]^2 + (y-5)^2}$$

$$(x-6)^2 + (y+5)^2 =$$

$$2^2[(x+4)^2 + (y-5)^2]$$

$$x^2 - 12x + 36 + y^2 + 10y + 25 =$$

$$4(x^2 + 8x + 16 + y^2 - 10y + 25)$$

$$x^2 - 12x + 36 + y^2 + 10y + 25 =$$

$$4x^2 + 32x + 64 + 4y^2 - 40y + 100$$

$$3x^2 + 3y^2 + 44x - 50y + 103 = 0$$

16 (a) (i) $y = -\frac{4}{3}x$

$$m_{OA} = -\frac{4}{3}$$

$$m_{AB} = -\frac{1}{\left(-\frac{4}{3}\right)} = \frac{3}{4}$$

$$4y = kx + 25$$

$$y = \frac{k}{4}x + \frac{25}{4}$$

$$m_{AB} = \frac{k}{4}$$

$$\therefore \frac{k}{4} = \frac{3}{4} \Rightarrow k = 3$$

(ii) $4y = 3x + 25 \dots (1)$

$$y = -\frac{4}{3}x \dots (2)$$

Substitute (2) into (1) :

$$4\left(-\frac{4}{3}x\right) = 3x + 25$$

$$-16x = 9x + 75$$

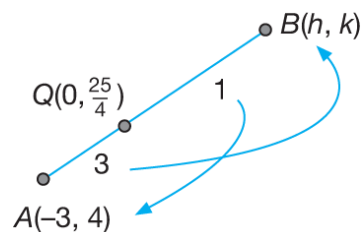
$$25x = -75$$

$$x = -3$$

$$\text{From (2) : } y = -\frac{4}{3}(-3) = 4$$

Hence, A is point $(-3, 4)$.

(b) (i)



Equating the x -coordinates,

$$\frac{1(-3) + 3h}{3+1} = 0$$

$$h = 1$$

Equating the y -coordinates,

$$\frac{1(4) + 3k}{3+1} = \frac{25}{4}$$

$$4 + 3k = 25$$

$$k = 7$$

Hence, B is point $(1, 7)$

(ii) $m_{CB} = m_{OA} = -\frac{4}{3}$

The equation of BC is

$$y - 7 = -\frac{4}{3}(x - 1)$$

$$3y - 21 = -4x + 4$$

$$3y = -4x + 25$$

(c) $PA = 2PB$

$$PA^2 = 2^2 PB^2$$

$$(x+3)^2 + (y-4)^2 = 2^2[(x-1)^2 + (y-7)^2]$$

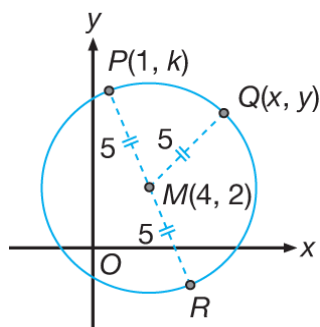
$$x^2 + 6x + 9 + y^2 - 8y + 16 =$$

$$4(x^2 - 2x + 1 + y^2 - 14y + 49)$$

$$x^2 + 6x + y^2 - 8y + 25 =$$

$$4x^2 - 8x + 4y^2 - 56y + 200$$

$$3x^2 - 14x + 3y^2 - 48y + 175 = 0$$



(a) Let Q be point (x, y) .

$$QM = 5$$

$$\sqrt{(x-4)^2 + (y-2)^2} = 5$$

$$(x-4)^2 + (y-2)^2 = 5^2$$

$$x^2 - 8x + 16 + y^2 - 4y + 4 = 25$$

$$x^2 - 8x + y^2 - 4y - 5 = 0$$

(b) (i) Since $P(1, k)$ lies on the locus of Q ,

$$1^2 - 8(1) + k^2 - 4k - 5 = 0$$

$$k^2 - 4k - 12 = 0$$

$$(k+2)(k-6) = 0$$

$$k = -2 \text{ or } 6$$

$k = -2$ is not accepted.

$$\therefore k = 6$$

(ii) Since the points P and R lie on the locus of Q , the distances of the points from point M are equal. This means that M is the midpoint of PR .

Let R be point (a, b) .

$$\left(\frac{1+a}{2}, \frac{6+b}{2}\right) = (4, 2)$$

By comparison,

$$\frac{1+a}{2} = 4 \quad \text{and} \quad \frac{6+b}{2} = 2$$

$$a = 7$$

$$b = -2$$

Hence, R is point $(7, -2)$.

(c) $O(0, 0)$, $P(1, 6)$, $R(7, -2)$

Area of $\triangle OPR$

$$= \frac{1}{2} \begin{vmatrix} 0 & 1 & 7 & 0 \\ 0 & 6 & -2 & 0 \end{vmatrix}$$

$$= \frac{1}{2} |-2 - 42|$$

$$\begin{aligned} &= \frac{1}{2} |-44| \\ &= |-22| \\ &= 22 \text{ units}^2 \end{aligned}$$

18 (a) (i) Area of $\triangle OAB$

$$= \frac{1}{2} \begin{vmatrix} 0 & 3 & 6 & 0 \\ 0 & -5 & 1 & 0 \end{vmatrix}$$

$$= \frac{1}{2} |-3 - (-30)|$$

$$= \frac{1}{2} |27|$$

$$= 13.5 \text{ units}^2$$

(ii) $AB = \sqrt{(6 - (-3))^2 + (1 - (-5))^2}$

$$= \sqrt{9^2 + 6^2}$$

$$= \sqrt{117}$$

$$= 10.82 \text{ units}$$

(b) (i) Let $P = (x, y)$

$$PA = 2PB$$

$$(PA)^2 = (2PB)^2$$

$$PA^2 = 4PB^2$$

$$(x - (-3))^2 + (y - (-5))^2 =$$

$$4((x - 6)^2 + (y - 1)^2)$$

$$(x + 3)^2 + (y + 5)^2 = 4((x - 6)^2 + (y - 1)^2)$$

$$x^2 + 6x + 9 + y^2 + 10y + 25 =$$

$$4(x^2 - 12x + 36 + y^2 - 2y + 1)$$

$$x^2 + 6x + 9 + y^2 + 10y + 25 =$$

$$4x^2 - 48x + 144 + 4y^2 - 8y + 4$$

$$0 = 3x^2 - 54x + 3y^2 - 18y + 114$$

$$x^2 - 18x + y^2 - 6y + 38 = 0$$

(ii) At the y -axis, $x = 0$.

$$\therefore 0^2 - 18(0) + y^2 - 6y + 38 = 0$$

$$y^2 - 6y + 38 = 0$$

$$b^2 - 4ac$$

$$= (-6)^2 - 4(1)(38)$$

$$= 36 - 152$$

$$= -116$$

Since $b^2 - 4ac < 0$, the quadratic equation does not have real roots.

Hence, the locus of P will not intersect the y -axis.

19 (a) (i) Gradient of the straight line
 $2x - y - 5 = 0 \Rightarrow y = 2x - 5$ is 2.

$$\therefore m_{BC} = 2$$

$$\therefore m_{AB} = -\frac{1}{m_{BC}} = -\frac{1}{2}$$

Hence, the equation of the straight line AB is

$$y - y_1 = m(x - x_1)$$

$$y - (-5) = -\frac{1}{2}[x - (-10)]$$

At point $A(-10, -5)$,
 $x_1 = -10, y_1 = -5$.

$$2(y + 5) = -(x + 10)$$

$$2y + 10 = -x - 10$$

$$2y = -x - 20$$

(ii) Equation of BC : $2x - y - 5 = 0$... (1)

Equation of AB : $x + 2y + 20 = 0$... (2)

$$4x - 2y - 10 = 0 \text{ ... (1) } \times 2$$

$$(+)\frac{x + 2y + 20 = 0 \text{ ... (2)}}{5x \quad +10 = 0}$$

$$\therefore x = -2$$

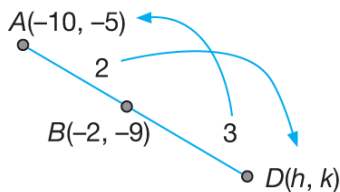
From (1):

$$2(-2) - y - 5 = 0$$

$$\therefore y = -9$$

Hence, B is point $(-2, -9)$.

(b)



$$B = (-2, -9)$$

$$\left(\frac{3(-10) + 2h}{2+3}, \frac{3(-5) + 2k}{2+3}\right) = (-2, -9)$$

$$\left(\frac{-30 + 2h}{5}, \frac{-15 + 2k}{5}\right) = (-2, -9)$$

Equating the x -coordinates:

$$\frac{-30 + 2h}{5} = -2$$

$$2h = 20$$

$$h = 10$$

Equating the y -coordinates:

$$\frac{-15 + 2k}{5} = -9$$

$$2k = -30$$

$$k = -15$$

Hence, D is point $(10, -15)$.

Area of $\triangle ADO$

$$= \frac{1}{2} \begin{vmatrix} -10 & 10 & 0 \\ -5 & -15 & 0 \\ 0 & 0 & -5 \end{vmatrix}$$

$$= \frac{1}{2} |150 - (-50)|$$

$$= \frac{1}{2} |200|$$

$$= 100 \text{ units}^2$$

(c) Let P be point (x, y) .

Since $\angle APB = 90^\circ$, AP is perpendicular to PB .

Hence, $(m_{AP})(m_{PB}) = -1$

$$\left(\frac{y - (-5)}{x - (-10)}\right)\left(\frac{y - (-9)}{x - (-2)}\right) = -1$$

$$\frac{(y + 5)(y + 9)}{(x + 10)(x + 2)} = -1$$

$$(y + 5)(y + 9) = -(x + 10)(x + 2)$$

$$y^2 + 14y + 45 = -(x^2 + 12x + 20)$$

$$x^2 + y^2 + 12x + 14y + 65 = 0$$

20 (a) (i) Radius of circle,

$$MA = \sqrt{(1+3)^2 + (3-0)^2}$$

$$MA = \sqrt{25}$$

$$MA = 5$$

$$MR = 5$$

$$\sqrt{(x-1)^2 + (y-3)^2} = 5$$

$$x^2 - 2x + 1 + y^2 - 6y + 9 = 5^2$$

$$x^2 + y^2 - 2x - 6y - 15 = 0$$

(ii) $4^2 + k^2 - 2(4) - 6k - 15 = 0$

$$16 + k^2 - 8 - 6k - 15 = 0$$

$$k^2 - 6k - 7 = 0$$

$$(k + 1)(k - 7) = 0$$

$$k = -1 \text{ or } k = 7$$

$k = -1$ is not accepted because the question states that $k > 0$.

$$\therefore k = 7$$

$$(b) \text{ Gradient } MA = \frac{3-0}{1+3} = \frac{3}{4}$$

$$\text{Hence, gradient } MC = -\frac{4}{3}$$

The equation of MC is

$$y-0 = -\frac{4}{3}(x+3)$$

$$3y = -4x - 12$$

$$\text{At } C, x = 0$$

$$3y = -4(0) - 12$$

$$y = -4$$

Hence, C is point $(0, -4)$.

Area of OAC

$$= \frac{1}{2} \begin{vmatrix} 0 & -3 & 0 & 0 \\ 0 & 0 & -4 & 0 \end{vmatrix}$$

$$= \frac{1}{2} |12|$$

$$= 6 \text{ units}^2$$

SPM Spot

$$1 (a) \text{ Gradient of the line } AMC = \frac{3-9}{-5-(-9)} \\ = -\frac{3}{2}$$

Gradient of the perpendicular bisector

$$= -\left(-\frac{2}{3}\right) = \frac{2}{3}$$

Equation of the perpendicular bisector is

$$y-3 = \frac{2}{3}[x-(-5)]$$

$$3y-9 = 2x+10$$

$$3y = 2x+19$$

(b) For point $B(-8, k)$,

$$3k = 2(-8)+19$$

$$3k = 3$$

$$k = 1$$

Thus, B is point $(-8, 1)$.

Let C be point (p, q) .

Equating the x -coordinates,

$$\frac{-9+p}{2} = -5$$

$$-9+p = -10$$

$$p = -1$$

Equating the y -coordinates,

$$\frac{9+q}{2} = 3$$

$$9+q = 6$$

$$q = -3$$

Thus, C is point $(-1, -3)$.

Area of $ABCD$

$$= \frac{1}{2} \begin{vmatrix} -9 & 8 & 1 & 9 \\ 9 & 1 & -3 & 9 \end{vmatrix}$$

$$= \frac{1}{2} |-9+24-9+9 - (-72-1-3-81)|$$

$$= \frac{1}{2} (172)$$

$$= 86 \text{ units}^2$$

(c) $AD = 9 + 1 = 10$ units

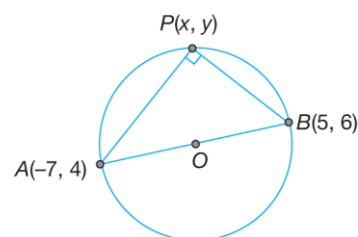
$$\sqrt{[x-(-9)]^2 + (y-9)^2} = 10$$

$$x^2 + 18x + 81 + y^2 - 18y + 81 = 100$$

Hence, the equation of the locus of P is

$$x^2 + y^2 + 18x - 18y + 62 = 0$$

2 (a) (i)



$$m_{AP} = \frac{y-4}{x+7}$$

$$m_{BP} = \frac{y-6}{x-5}$$

The angle at the circumference of a semicircle is always 90° .

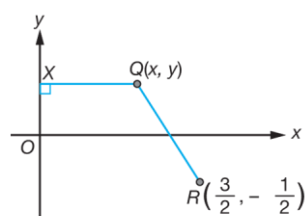
$$\left(\frac{y-4}{x+7}\right)\left(\frac{y-6}{x-5}\right) = -1$$

$$(y-4)(y-6) = -(x+7)(x-5)$$

$$y^2 - 10y + 24 = -(x^2 + 2x - 35)$$

$$y^2 - 10y + x^2 + 2x - 11 = 0$$

(ii)



$$\sqrt{\left(x - \frac{3}{2}\right)^2 + \left(y + \frac{1}{2}\right)^2} = 2.5x$$

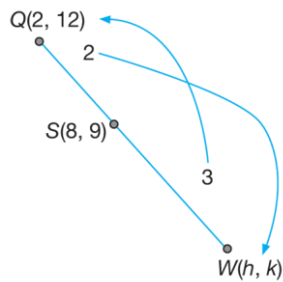
$$\left(x - \frac{3}{2}\right)^2 + \left(y + \frac{1}{2}\right)^2 = 6.25x^2$$

$$x^2 - 3x + \frac{9}{4} + y^2 + y + \frac{1}{4} = 6.25x^2$$

$$4x^2 - 12x + 9 + 4y^2 + 4y + 1 = 25x^2$$

$$21x^2 + 12x - 4y^2 - 4y - 10 = 0$$

(b)



(i) The coordinates of point S are

$$\left(\frac{3(2) + 2h}{2 + 3}, \frac{3(12) + 2k}{2 + 3}\right), \text{ i.e.}$$

$$\left(\frac{6 + 2h}{5}, \frac{36 + 2k}{5}\right).$$

But it is given that the coordinates of point S are $(8, 9)$.

By comparison,

$$\frac{6 + 2h}{5} = 8$$

$$6 + 2h = 40$$

$$2h = 34$$

$$h = 17$$

$$\frac{36 + 2k}{5} = 9$$

$$36 + 2k = 45$$

$$2k = 9$$

$$k = 4.5$$

Hence, the coordinates of the point W are $(17, 4.5)$.

(ii) Area of the quadrilateral $PSRQ = 45$ units²

$$\frac{1}{2} \begin{vmatrix} 0 & 8 & 8 & 2 & 0 \\ y & 9 & 16 & 12 & y \end{vmatrix} = 45$$

$$128 + 96 + 2y - (8y + 72 + 32) = 90$$

$$120 - 6y = 90$$

$$6y = 30$$

$$y = 5$$

Hence, the y -intercept of the straight line PQ is 5 .