Form 5 Chapter 4
Permutation and Combination
Fully-Worked Solutions

UPSKILL 4.1a

1 Number of ways to travel from town *P* to *R* via town *Q*
$= 2\times5 = 10$

2 Number of ways to travel to Butterworth to Kuala Lumpur via Ipoh by taking a bus
$= 4\times5 = 20$

3 Number of ways to match a blouse, a gown and a pair of shoes
$= 5\times4\times2$
$= 40$

UPSKILL 4.1b

1 (a) $9! = 9\times8\times7\times6\times5\times4\times3\times2\times1$
$= 362\ 880$

(b) $\dfrac{n!\,0!}{(n-1)!}$

$= \dfrac{n(n-1)\ldots}{(n-1)\ldots}$

$= n$

2 Number of arrangements
$= 6!$
$= 720$

3 Number of ways
$= 7!$
$= 5\ 040$

4 Number of ways
$= 6!$
$= 720$

5 Number of 5-digit numbers
$= 5!$
$= 120$

6 (a) (i) Number of arrangements
$= 5!$
$= 120$

(ii) Number of arrangements
$= 7!$
$= 5\ 040$

(b) (i) <u>E</u> _ _ _ _ … 4!

<u>U</u> _ _ _ _ … 4!

Number of arrangements
$= 4!\times2$
$= 48$

(ii) <u>A</u> _ _ _ _ _ _ … 6!

<u>U</u> _ _ _ _ _ _ … 6!

Number of arrangements
$= 6!\times2$
$= 1\ 440$

7 _ _ _ <u>4</u> … 3!

_ _ _ <u>6</u> … 3!

Number of 4-digit even numbers
$= 3!\times 2$
$= 12$

8 _ _ _ <u>1</u> … 3!

_ _ _ <u>7</u> … 3!

Number of 4-digit odd numbers
$= 3!\times 2$
$= 12$

9 <u>3</u> _ _ _ … 3!

<u>4</u> _ _ _ … 3!

Number of 4-digit numbers greater than 3 000
$= 3!\times 2$
$= 12$

10 If the vowels have to be side-by-side, they are counted as I object. Along with the other 5 objects, there are altogether 6 objects.

OUE	*C*	*M*	*P*	*T*	*R*
√	√	√	√	√	√

This gives 6! = 720.

But the vowels can also be arranged among themselves

O	*U*	*E*
√	√	√

This gives 3! = 6.
Using the multiplication rule, the number of arrangements
= 720 × 6
= 4 320

11 (a) Number of arrangements

$= \frac{8!}{4!\,2!} = 840$

(b) Number of arrangements

$= \frac{10!}{2!\;2!\,2!} = 453\,600$

12 Number of arrangements
$= 3! \times 6! \times 4! \times 2!$
= 207 360

13 If the letters *O* and *E* are to be side-by-side, they are counted as 1 object. Along with the other 5 letters, they are altogether 6 objects.

OE	*S*	*U*	*P*	*P*	*S*
√	√	√	√	√	√

This gives $\frac{6!}{2!\,2!} = 180$

But *O* and *E* can also be arranged among themselves.

O	*E*
√	√

This gives 2! = 2.

Using the multiplication rule, the number of arrangements
= 180 × 2
= 360

14 The number of arrangements if no restriction is imposed

$= \frac{12!}{2!\,3!\,2!} = 19\,958\,400$

If the 5 vowels are to be side-by-side, they are counted as 1 object. Together with the 7 consonants, there are altogether 8 objects.

EEAIO	*T*	*S*	*S*	*S*	*L*	*L*	*T*	*N*
√	√	√	√	√	√	√	√	√

This gives $\frac{8!}{3!\,2!} = 3\,360$

But the 5 vowels can also be arranged among themselves.

E	*E*	*A*	*I*	*O*
√	√	√	√	√

This gives $\frac{5!}{2!} = 60$.

Using the multiplication rule, the number of arrangements
= 3 360 × 60
= 201 600

15 Number of arrangements if no restriction is imposed

$= \frac{12!}{2!\,3!\,2!} = 19\,958\,400$

If the 4 vowels have to be side-by-side, they are counted as 1 object. Together with the 8 consonants, there are altogether 9 objects.

OEIO	*C*	*R*	*S*	*S*	*S*	*C*	*T*	*N*
√	√	√	√	√	√	√	√	√

This gives $\frac{9!}{3!\,2!} = 30\,240$

But the 4 vowels can also be arranged among themselves.

O	*E*	*I*	*O*
√	√	√	√

This gives $\frac{4!}{2!} = 12$.

Using the multiplication rule, the number of arrangements
= 30 240 × 12
= 362 880

16 The number of arrangements in a circle
= (8 – 1)! = 5 040

UPSKILL 4.1c

1 $^9P_6 = \dfrac{9!}{(9-6)!} = 60\ 480$

2 Number of 4-digit numbers
= $^5P_3$
= 60

3 _ _ _ 1

_ _ _ 3

_ _ _ 5

_ _ _ 7

Number of 4-digit odd numbers
= $^6P_3 \times 4$
= 480

4 The number of numbers that can be formed
= $^6P_1 + {}^6P_2 + {}^6P_3 + {}^6P_4 + {}^6P_5 + {}^6P_6$
= 1 956

5 _ _ _ _ 2 … 5!

_ _ _ _ 4 … 5!

_ _ _ _ 6 … 5!

_ _ _ _ 8 … 5!

Number of 5-digit even numbers
$= {}^8P_4 \times 4 = 6\ 720$

6 The number of numbers that can be formed
= $^5P_1 + {}^5P_2 + {}^5P_3 + {}^5P_4 + 5P_5$
= 325

7 (a) (i) Number of arrangements
= $^5P_3$
= 60

(ii) Number of arrangements
= $^8P_3$
= 336

(b) (i) *O* _ _

E _ _

Number of arrangements
= $^4P_2 \times 2$
= 24

(ii) *U* _ _ … $^7P_2$

A _ _ … $^7P_2$

Number of arrangements
= $^7P_2 \times 2$
= 84

8 $^4P_2 \times 6! + {}^4P_2 \times 6!$

Two children at the left of the table.

Two children at the right of the table.

= 17 280

9 (a) *W* *W* *W* *W* *W*

M *M* *M* *M*

Number of arrangements
= 5! × 4!
= 2 880

(b) Jika the 4 men want to sit together, they will be counted as 1 object. Together with the 5 women, there are altogether 6 objects.

LLLL	*W*	*W*	*W*	*W*	*W*
√	√	√	√	√	√

This gives 6!.

But the 4 men can be arranged among themselves.

L	*L*	*L*	*L*
√	√	√	√

This gives 4!.

Using the multiplication rule, the number of arrangements
= 6! × 4!
= 17 280

10 If the 3 girls want to sit together, they will be counted as 1 object. Together with the 4 boys, there are altogether 5 objects.

GGG	*B*	*B*	*B*	*B*
√	√	√	√	√

This gives 5!.

But the 3 girls can also be arranged among themselves.

G	*G*	*G*
√	√	√

This gives 3!.

Using the multiplication rule, the number of arrangements
= 5! × 3!
= 720

11 Number of arrangements
= 3! × 5! × 4! × 3!
= 103 680

12 Number of arrangements
= 4! × 3! × 4! × 2
= 6 912

UPSKILL 4.2

1 (a) $^9C_5 = \dfrac{9!}{5! \times (9-5)!} = 126$

(b) $^6C_4 = \dfrac{6!}{4! \times (6-4)!} = 15$

(c) $^8C_3 = \dfrac{8!}{3! \times (8-3)!} = 56$

(d) $^{10}C_6 = \dfrac{10!}{6! \times (10-6)!} = 210$

3 $^8C_2 = 28$

4 (a) $^9C_6 = 84$

(b) $^5C_4 \times {}^4C_2 = 30$

5 Number of committees
$= {}^9C_5 \times {}^7C_4$
= 4 410

6

HOT TIPS
Three point along a straight line cannot form a triangle.

Number of triangles that can be formed
$= {}^{10}C_3 - {}^4C_3 - {}^6C_3 = 120 - 4 - 20 = 96$

7 (a) Number of committees
$= {}^1C_1 \times {}^{12}C_6$
= 924

(b) Number of committees
$= {}^8C_5 \times {}^5C_2$
= 560

(c)

	Males	**Females**	**Number of choices**
Available	8	5	
Required	4	3	$^8C_4 \times {}^5C_3$
	3	4	$^8C_3 \times {}^5C_4$
	2	5	$^8C_2 \times {}^5C_5$
	1	6	Impossible
	0	7	Impossible

Number of committees
$= {}^8C_4 \times {}^5C_3 + {}^8C_3 \times {}^5C_4 + {}^8C_2 \times {}^5C_5$
= 700 + 280 + 28
= 1 008

8 (a) Number of ways
$= {}^{12}C_5 \times {}^{10}C_7$
= 95 040

(b) Number of ways
$= {}^9C_4 \times {}^{13}C_8$
= 162 162

(c) Number of ways
$= {}^4C_3 \times {}^5C_3 \times {}^6C_3 \times {}^7C_3$
= 28 000

9 (a) Number of committees

$= {}^{10}C_6$

$= 210$

(b)

	Teacher	**Student**	**Number of choices**
Available	4	6	
Required	2	4	${}^4C_2 \times {}^6C_4$
	1	5	${}^4C_1 \times {}^6C_5$
	0	6	${}^4C_0 \times {}^6C_6$

Number of ways

$= {}^4C_2 \times {}^6C_4 + {}^4C_1 \times {}^6C_5 + {}^4C_0 \times {}^6C_6$

$= 90 + 24 + 1$

$= 115$

10 Number of choices

$= {}^6C_1 + {}^6C_2 + {}^6C_3 + {}^6C_4 + {}^6C_5 + {}^6C_6$

$= 63$

Summative Practice 4

1 Number of ways

$= 5 \times 6$

$= 30$

2 Number of permutations

$= 6!$

$= 720$

3 3 _ _ 1 … 2!

4 _ _ 1 … 2!

4 _ _ 3 … 2!

Number of ways

$= 2! \times 3$

$= 6$

4 (a) Number of numbers = 6! = 720

(b) (i) 5 _ _ _ _ _ … 5!

6 _ _ _ _ _ … 5!

Number of numbers

$= 5! \times 2$

$= 240$

(ii) 1 _ _ _ _ 2 … 4!

1 _ _ _ _ 4 … 4!

1 _ _ _ _ 6 … 4!

Number of numbers

$= 4! \times 3$

$= 72$

5 5 _ _ _ … ${}^4P_3$

6 _ _ _ … ${}^4P_3$

7 _ _ _ … ${}^4P_3$

Number of numbers

$= {}^4P_3 \times 3$

$= 72$

6 *E* _ _ _ … ${}^5P_3$

I _ _ _ … ${}^5P_3$

U _ _ _ … ${}^5P_3$

Number of arrangements

$= {}^5P_3 \times 3$

$= 180$

7 The prime numbers are 2, 3, 5 and 7.

2 _ _ _ … 3!

3 _ _ _ … 3!

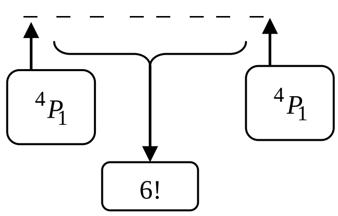
5 _ _ _ … 3!

Number of numbers

$= 3! \times 3$

$= 18$

8



Number of arrangements

$= {}^4P_1 \times {}^4P_1 \times 6!$

$= 11\,520$

9 (a) Number of arrangements without restrictions

$= \dfrac{10!}{5!\ 2!} = 15\ 120$

(b) If the 5 vowels have to be side-by-side, there are counted as 1 object. Together with the 7 consonants, there are altogether 8 objects.

EAE	*S*	*S*	*S*	*S*	*S*	*L*	*F*
√	√	√	√	√	√	√	√

This gives $\dfrac{8!}{5!} = 336$

But the 5 vowels can also be arranged among themselves.

E	*A*	*E*
√	√	√

This gives $\dfrac{3!}{2!} = 3$.

Using the multiplication rule, the number of arrangements

$= 336 \times 3$

$= 1\ 008$

10 Number of ways $= {}^{8}C_{3} = 56$

11 Number of teams

$= {}^{8}C_{5} \times {}^{6}C_{4}$

$= 840$

12 (a) Number of triangles

$= {}^{8}C_{3} - {}^{6}C_{3}$

$= 36$

(b) Number of triangles (with point *B* only)

$= {}^{6}C_{2} = 15$

Number of triangles (without points *A* and *B*) $= {}^{6}C_{1} = 6$

Total number of triangles

$= 15 + 6$

$= 21$

13 Number of ways

$= {}^{8}C_{4} \times {}^{4}C_{3} \times {}^{10}C_{4}$

$= 58\ 800$

14 Number of combinations

$= {}^{2}C_{2} \times {}^{10}C_{6}$

$= 210$

15 Number of combinations

$= {}^{1}C_{1} \times {}^{10}C_{4} \times {}^{6}C_{6}$

$= 210$

16 Number of choices

$= {}^{10}C_{3} \times {}^{7}C_{5} \times {}^{2}C_{2}$

$= 2\ 520$

17 (a) Number of committees

$= {}^{13}C_{6}$

$= 1\ 716$

(b) Number of committees

$= {}^{7}C_{3} \times {}^{6}C_{3} + {}^{7}C_{4} \times {}^{6}C_{2}$

$= 1\ 225$

18 (a) Number of ways

$= {}^{5}C_{2} \times {}^{7}C_{4}$

$= 350$

(b)

	Stationery	**Story book**	**Number of choices**
Available	5	7	
Required	3	3	${}^{5}C_{3} \times {}^{7}C_{3}$
	4	2	${}^{5}C_{4} \times {}^{7}C_{2}$
	5	1	${}^{5}C_{5} \times {}^{7}C_{1}$
	6	0	Impossible

Number of ways

$= {}^{5}C_{3} \times {}^{7}C_{3} + {}^{5}C_{4} \times {}^{7}C_{2} + {}^{5}C_{5} \times {}^{7}C_{1}$

$= 350 + 105 + 7$

$= 462$

19 (a) Number of arrangements

$= {}^{5}P_{3}$

$= 60$

(b) Number of combinations

$= {}^{5}C_{2}$

$= 10$

20 (a) Number of teams

$= {}^{10}C_3$

$= 120$

(b)

	Male	Female	Number of choices
Available	4	6	
Required	2	1	${}^4C_2 \times {}^6C_1$
	1	2	${}^4C_1 \times {}^6C_2$
	0	3	${}^4C_0 \times {}^6C_3$

Number of teams

$= {}^4C_2 \times {}^6C_1 + {}^4C_1 \times {}^6C_2 + {}^4C_0 \times {}^6C_3$

$= 36 + 60 + 20$

$= 116$

SPM Spot

1 If the vowels have to be side by side, they will be counted as 1 object. Together with the consonants, there are 6 objects.

[*I I U I*] *N S T T S*

√ √ √ √ √ √

This gives $\frac{6!}{2!2!} = 180$.

But the vowels can also be arranged among themselves.

[*I I U I*
√ √ √ √]

This gives $\frac{4!}{3!} = 4$.

Hence, using the multiplication rule, the total number of arrangements is $180 \times 4 = 720$.

2

	Girl	Boy	Number of choices
Available	3	2	
Required	1	2	${}^3C_1 \times {}^2C_2$
	2	1	${}^3C_2 \times {}^2C_1$
	3	0	${}^3C_3 \times {}^2C_0$

Hence, the total number of choices

$= {}^3C_1 \times {}^2C_2 + {}^3C_2 \times {}^2C_1 + {}^3C_3 \times {}^2C_0$

$= 3 + 6 + 1$

$= 10$

3 (a) (i) Number of arrangements without restriction $= 6! = 720$

(ii)

[4 6] 3 5 7 9

√ √ √ √ √

If the even numbers 4 and 6 have to be side by side, the two even numbers is considered as 1 object. Together with the numbers 3, 5, 7 and 9, there are 5 objects. This gives 5! = 120.

But the even numbers 2 and 6 can interchange positions. This gives 2! = 2.

Using the multiplication rule, the number of different arrangements

$= 120 \times 2$

$= 240$

Hence, the number of different arrangements if the even numbers 4 and 6 cannot be side by side

$= 720 – 240$

$= 480$

(b)

	Mathematics books	Science books	Number of choices
Available	6	6	
Number of books that can be bought	4	4	${}^6C_4 \times {}^6C_4$
	3	5	${}^6C_3 \times {}^6C_5$
	2	6	${}^6C_2 \times {}^6C_6$
	1	7	Impossible
	0	8	Impossible

Hence, the number of different ways that the student could buy the books

$= \left({}^6C_4 \times {}^6C_4\right) + \left({}^6C_3 \times {}^6C_5\right) + \left({}^6C_2 \times {}^6C_6\right)$

$= 225 + 120 + 15$

$= 360$