Form 5: Chapter 2
Matrices
Fully-worked Solutions

UPSKILL 2.1

1 The matrix is $\begin{bmatrix} 15 & 10 & 5 \\ 12 & 11 & 2 \\ 10 & 13 & 3 \end{bmatrix}$.

2 The matrix is $\begin{bmatrix} 5 & 3 \\ 2 & 4 \end{bmatrix}$.

3

	Number of rows	*Number of columns*	*Order*
(a)	1	3	1×3
(b)	3	2	3×2
(c)	2	2	2×2
(d)	3	3	3×3

4 (a) Column matrix
(b) Row matrix
(c) Square matrix
(d) Rectangular matrix

5

a_{11}	a_{12}	a_{21}	a_{22}
7	5	2	–1
3	–5	4	0
12	10	9	11
2	–4	–2	–6

6 $e = b_{12} = -8$
$f = b_{23} = 7$
$g = a_{41} = -3$
$h = a_{31} = -6$

7 (a) Same
(b) Not the same

8 (a) $\begin{bmatrix} -2 & 4h-1 \\ 3k-5 & -7 \end{bmatrix} = \begin{bmatrix} -2 & 15 \\ -11 & -7 \end{bmatrix}$

$4h - 1 = 15$
$4h = 16$
$h = 4$

$3k - 5 = -11$
$3k = -6$
$k = -2$

(b) $\begin{bmatrix} 9-h & -5 \\ 8 & h+3 \\ 2k-1 & -6 \end{bmatrix} = \begin{bmatrix} 10 & -5 \\ 8 & h+3 \\ 3k+2 & -6 \end{bmatrix}$

$9 - h = 10$
$h = -1$

$2k - 1 = 3k + 2$
$k = -3$

(c) $\begin{bmatrix} h+3k & -6 & h-17 \end{bmatrix} = \begin{bmatrix} -3 & -6 & 2k \end{bmatrix}$

$h + 3k = -3 \;\ldots (1)$

$h - 17 = 2k$
$h - 2k = 17 \;\ldots (2)$

$(1) - (2):\; 5k = -20$
$k = -4$

From (1) :
$h + 3(-4) = -3$
$h = 12 - 3$
$h = 9$

(d) $\begin{bmatrix} -1 \\ 13-k \\ 4h-3k \end{bmatrix} = \begin{bmatrix} -1 \\ 8h \\ 17 \end{bmatrix}$

$13 - k = 8h$
$k = 13 - 8h \;\ldots (1)$

$4h - 3k = 17 \;\ldots (2)$

Substitute (1) into (2) :
$4h - 3(13 - 8h) = 17$
$4h - 39 + 24h = 17$
$28h = 56$
$h = 2$

From (1) :
$k = 13 - 8(2) = -3$

9 (a) $\begin{bmatrix} 3 & 2a+b \\ 2a-b & 1 \end{bmatrix} = \begin{bmatrix} 3 & 8 \\ 2c & 2b-a \end{bmatrix}$

$2a + b = 8$
$b = 8 - 2a \;\ldots (1)$

$-a + 2b = 1 \;\ldots (2)$

Substitute (1) into (2) :
$-a + 2(8 - 2a) = 1$
$-a + 16 - 4a = 1$
$-5a = -15$
$a = 3$

From (1) :
$b = 8 - 2(3)$ … (1)
$b = 2$

$$2a - b = 2c$$
$$2(3) - 2 = 2c$$
$$2c = 4$$
$$c = 2$$

(b) $\begin{bmatrix} a-2b & -5 & \frac{abc}{4} \end{bmatrix} = \begin{bmatrix} 6 & b-a & -2 \end{bmatrix}$

$a - 2b = 6$
$a = 2b + 6$ … (1)

$b - a = -5$ … (2)

Substitute (1) into (2) :
$b - (2b + 6) = -5$
$-b - 6 = -5$
$b = -6 + 5$
$b = -1$

From (1) :
$a = 2(-1) + 6 = 4$

$$\frac{abc}{4} = -2$$
$$\frac{(4)(-1)(c)}{4} = -2$$
$$-4c = -8$$
$$c = 2$$

UPSKILL 2.2

1 (a) Yes (b) No

2 (a) $\begin{bmatrix} -2 & 4 \\ -5 & -5 \end{bmatrix} + \begin{bmatrix} -8 & -10 \\ -3 & 7 \end{bmatrix} = \begin{bmatrix} -10 & -6 \\ 2 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} -2 & 3 \\ 7 & -8 \\ -12 & 6 \end{bmatrix} + \begin{bmatrix} 6 & -10 \\ -16 & 5 \\ -14 & 4 \end{bmatrix} = \begin{bmatrix} 4 & -7 \\ -9 & -3 \\ -26 & 10 \end{bmatrix}$

3 (a) $\begin{bmatrix} 1 & -2 & 3 \\ -1 & 5 & -3 \end{bmatrix} - \begin{bmatrix} -5 & 9 & -7 \\ 8 & -6 & 4 \end{bmatrix}$
$= \begin{bmatrix} 6 & -11 & 10 \\ -9 & 11 & -7 \end{bmatrix}$

(b) $\begin{bmatrix} -9 & 12 & 2 \\ 6 & -4 & 7 \\ -1 & -13 & 8 \end{bmatrix} - \begin{bmatrix} 11 & -2 & -3 \\ -12 & -5 & 1 \\ 5 & -4 & -2 \end{bmatrix}$
$= \begin{bmatrix} -20 & 14 & 5 \\ 18 & 1 & 6 \\ -6 & -9 & 10 \end{bmatrix}$

4 (a) $\begin{bmatrix} 2a \\ -b \end{bmatrix} - \begin{bmatrix} -3a \\ -5b \end{bmatrix} = \begin{bmatrix} 5a \\ 4b \end{bmatrix}$

(b) $\begin{bmatrix} -4c & -9d \end{bmatrix} + \begin{bmatrix} 9c & -2d \end{bmatrix} = \begin{bmatrix} 5c & -11d \end{bmatrix}$

(c) $\begin{bmatrix} 3e & 2f \\ -4f & 7e \end{bmatrix} - \begin{bmatrix} -2e & -f \\ -5f & 7e \end{bmatrix} = \begin{bmatrix} 5e & 3f \\ f & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 9g & -3h & -4i \end{bmatrix} - \begin{bmatrix} -2g & 3h & -5i \end{bmatrix}$
$= \begin{bmatrix} 11g & -6h & i \end{bmatrix}$

5 (a) $\begin{bmatrix} -2 \\ 3 \\ 8 \\ -12 \end{bmatrix} + \begin{bmatrix} 6 \\ -5 \\ -7 \\ 11 \end{bmatrix} - \begin{bmatrix} 0 \\ -4 \\ -3 \\ 13 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 4 \\ -14 \end{bmatrix}$

(b) $\begin{bmatrix} -2 & 5 \\ 3 & -4 \\ 3 & 6 \end{bmatrix} - \begin{bmatrix} 7 & 9 \\ -2 & 2 \\ 3 & 1 \end{bmatrix} + \begin{bmatrix} 12 & 8 \\ 18 & 11 \\ -9 & 10 \end{bmatrix}$
$= \begin{bmatrix} 3 & 4 \\ 23 & 5 \\ -9 & 15 \end{bmatrix}$

6 (a) $\begin{bmatrix} -3x & 3y \end{bmatrix} + \begin{bmatrix} 2x & -7y \end{bmatrix} - \begin{bmatrix} 5x & 9y \end{bmatrix}$
$= \begin{bmatrix} -6x & -13y \end{bmatrix}$

(b) $\begin{bmatrix} p \\ 4q \\ 2r \end{bmatrix} - \begin{bmatrix} -2p \\ -5q \\ 7r \end{bmatrix} + \begin{bmatrix} 6p \\ -10q \\ -4r \end{bmatrix} = \begin{bmatrix} 9p \\ -q \\ -9r \end{bmatrix}$

(c) $\begin{bmatrix} 3a & 5b \\ -6c & -d \end{bmatrix} - \begin{bmatrix} 2a & -2b \\ 4c & -7d \end{bmatrix} - \begin{bmatrix} 6a & 3b \\ 5c & -8d \end{bmatrix}$
$= \begin{bmatrix} -5a & 4b \\ -15c & 14d \end{bmatrix}$

7 (a) $\begin{bmatrix} x & 5 \\ 3 & -9 \end{bmatrix} + \begin{bmatrix} -3 & -4 \\ 6 & y \end{bmatrix} = \begin{bmatrix} 7 & 1 \\ 9 & -4 \end{bmatrix}$
$x - 3 = 7$
$x = 10$

$y - 9 = -4$
$y = 5$

(b) $\begin{bmatrix} 7 & 2 \\ -8 & y \end{bmatrix} + \begin{bmatrix} x & -4 \\ 6 & 3 \end{bmatrix} = \begin{bmatrix} 12 & 6 \\ -14 & 7 \end{bmatrix}$
$7 - x = 12$
$x = -5$

$$y-3=7$$
$$y=10$$

(c) $\begin{bmatrix}5 & 1\\ -7 & 3\end{bmatrix}+\begin{bmatrix}3 & x\\ y & 5\end{bmatrix}=\begin{bmatrix}8 & y\\ -2x & 8\end{bmatrix}$

$1+x=y \Rightarrow x-y=-1 \ \ldots (1)$

$-7+y=-2x \Rightarrow 2x+y-7 \ \ldots (2)$

(1) + (2) : $3x=6 \Rightarrow x=2$

From (1) : $2-y=-1 \Rightarrow y=3$

(d) $\begin{bmatrix}x & -8\\ 5 & 3x\end{bmatrix}-\begin{bmatrix}3y & 5\\ -9 & y\end{bmatrix}=\begin{bmatrix}11 & -13\\ 14 & 9\end{bmatrix}$

$x-3y=11$

$x=3y+11 \ \ldots (1)$

$3x-y=9 \ \ldots (2)$

Substitute (1) into (2) :

$3(3y+11)-y=9$

$9y+33-y=9$

$8y=-24$

$y=-3$

From (1) : $x=3(-3)+11$

$x=2$

8 (a) $\frac{1}{5}\begin{bmatrix}-15 & -35\\ 20 & 25\end{bmatrix}=\begin{bmatrix}-3 & -7\\ 4 & 5\end{bmatrix}$

(b) $\frac{1}{7}\begin{bmatrix}14 & -14 & 28\\ -35 & 42 & -49\\ 21 & -63 & 56\end{bmatrix}$

$=\begin{bmatrix}2 & -2 & 4\\ -5 & 6 & -7\\ 3 & -9 & 8\end{bmatrix}$

9 (a) $\begin{bmatrix}24 & -28\\ -16 & 12\end{bmatrix}$

$=4\begin{bmatrix}6 & -7\\ -4 & 3\end{bmatrix}$

(b) $\begin{bmatrix}24 & -48\\ -6 & -30\\ 18 & 12\end{bmatrix}$

$=6\begin{bmatrix}4 & -8\\ -1 & -5\\ 3 & 2\end{bmatrix}$

10 (a) $\begin{bmatrix}4 & -1 & \frac{2}{3}\\ \frac{5}{3} & \frac{8}{3} & -2\end{bmatrix}=\frac{1}{3}\begin{bmatrix}12 & -3 & 2\\ 5 & 8 & -6\end{bmatrix}$

$\therefore k=\frac{1}{3}$

(b) $\begin{bmatrix}-4\\ \frac{2}{5}\\ 3\\ -1\end{bmatrix}=\frac{1}{5}\begin{bmatrix}-20\\ 2\\ 15\\ -5\end{bmatrix}$

$\therefore k=\frac{1}{5}$

11 (a) $3\begin{bmatrix}-2\\ 0\\ -4\end{bmatrix}-2\begin{bmatrix}-3\\ -2\\ -5\end{bmatrix}+5\begin{bmatrix}-2\\ 1\\ 3\end{bmatrix}$

$=\begin{bmatrix}-6\\ 0\\ -12\end{bmatrix}-\begin{bmatrix}-6\\ -4\\ -10\end{bmatrix}+\begin{bmatrix}-10\\ 5\\ 15\end{bmatrix}$

$=\begin{bmatrix}-10\\ 9\\ 13\end{bmatrix}$

(b) $\frac{1}{4}\begin{bmatrix}-14 & 16\\ 12 & -8\end{bmatrix}+\frac{1}{9}\begin{bmatrix}18 & -27\\ -36 & 45\end{bmatrix}$

$=\begin{bmatrix}-1 & 4\\ 3 & -2\end{bmatrix}+\begin{bmatrix}2 & -3\\ -4 & 5\end{bmatrix}$

$=\begin{bmatrix}1 & 1\\ -1 & 3\end{bmatrix}$

(c) $\frac{1}{6}\begin{bmatrix}12 & -24\\ -18 & 30\end{bmatrix}+\frac{1}{3}\begin{bmatrix}3 & 9\\ 12 & -15\end{bmatrix}-\frac{2}{5}\begin{bmatrix}10 & -15\\ 20 & -25\end{bmatrix}$

$=\begin{bmatrix}2 & -4\\ -3 & 5\end{bmatrix}+\begin{bmatrix}1 & 3\\ 4 & -5\end{bmatrix}-\begin{bmatrix}4 & -6\\ 8 & -10\end{bmatrix}$

$=\begin{bmatrix}-1 & 5\\ -7 & 10\end{bmatrix}$

(d) $2\begin{bmatrix}1 & 2 & 3\\ 4 & 3 & 2\\ -2 & -1 & -3\end{bmatrix}+4\begin{bmatrix}3 & 2 & 1\\ -1 & 3 & 4\\ 2 & 3 & 1\end{bmatrix}$

$-3\begin{bmatrix}5 & 2 & 1\\ -1 & -2 & -3\\ 0 & 2 & 3\end{bmatrix}$

$=\begin{bmatrix}2 & 4 & 6\\ 8 & 6 & 4\\ -4 & -2 & -6\end{bmatrix}+\begin{bmatrix}12 & 8 & 4\\ -4 & 12 & 16\\ 8 & 12 & 4\end{bmatrix}$

$-\begin{bmatrix}15 & 6 & 3\\ -3 & -6 & -9\\ 0 & 6 & 9\end{bmatrix}$

$$= \begin{bmatrix} -1 & 6 & 7 \\ 7 & 24 & 29 \\ 4 & 4 & -11 \end{bmatrix}$$

12 (a) $4\begin{bmatrix} 2x \\ y \end{bmatrix} + 3\begin{bmatrix} x \\ 3y \end{bmatrix} - \frac{1}{5}\begin{bmatrix} 20x \\ -60y \end{bmatrix}$

$= \begin{pmatrix} 8x \\ 4y \end{pmatrix} + \begin{pmatrix} 3x \\ 9y \end{pmatrix} - \begin{pmatrix} 4x \\ -12y \end{pmatrix}$

$= \begin{bmatrix} 7x \\ 25y \end{bmatrix}$

(b) $5[3m \quad 2n] - 4[2m \quad 3n] + \frac{1}{4}[-16 \quad 24n]$

$= [15m \quad 10n] - [8m \quad 12n] + [-4m \quad 6n]$

$= [3m \quad 4n]$

(c) $\frac{1}{3}\begin{bmatrix} -3a & 6b \\ 9c & 15d \end{bmatrix} + \frac{1}{4}\begin{bmatrix} -8a & 12b \\ -16c & 20d \end{bmatrix}$

$- \frac{1}{6}\begin{bmatrix} 6a & -18b \\ 24c & -30d \end{bmatrix}$

$= \begin{bmatrix} -a & 2b \\ 3c & 5d \end{bmatrix} + \begin{bmatrix} -2a & 3b \\ -4c & 5d \end{bmatrix} - \begin{bmatrix} a & -3b \\ 4c & -5d \end{bmatrix}$

$= \begin{bmatrix} -4a & 8b \\ -5c & 15d \end{bmatrix}$

13 (a) $-3\begin{bmatrix} 3h & -1 \\ 2 & k \end{bmatrix} + 4\begin{bmatrix} 5 & -2 \\ 4 & 2k \end{bmatrix} = 5\begin{bmatrix} -5 & -1 \\ 2 & -3 \end{bmatrix}$

$\begin{bmatrix} -9h & 3 \\ -6 & -3k \end{bmatrix} + \begin{bmatrix} 20 & -8 \\ 16 & 8k \end{bmatrix} = \begin{bmatrix} -25 & -5 \\ 10 & -15 \end{bmatrix}$

$-9h + 20 = -25$

$-9h = -45$

$h = 5$

$-3k + 8k = -15$

$5k = -15$

$k = -3$

(b) $2\begin{bmatrix} 3 & 2k \\ 2 & 3h \end{bmatrix} + 3\begin{bmatrix} 3 & 2 \\ 7 & 4 \end{bmatrix} = 5\begin{bmatrix} 3 & 2 \\ 5 & 6 \end{bmatrix}$

$\begin{bmatrix} 6 & 4k \\ 4 & 6h \end{bmatrix} + \begin{bmatrix} 9 & 6 \\ 21 & 12 \end{bmatrix} = \begin{bmatrix} 15 & 10 \\ 25 & 30 \end{bmatrix}$

$4k + 6 = 10$

$k = 1$

$6h + 12 = 30$

$6h = 18$

$h = 3$

(c) $3\begin{bmatrix} h & 6 \\ -4 & 10 \end{bmatrix} + \frac{1}{3}\begin{bmatrix} -12 & 21 \\ k & 15 \end{bmatrix} = 5\begin{bmatrix} -2 & 5 \\ -1 & 7 \end{bmatrix}$

$\begin{bmatrix} 3h & 18 \\ -12 & 30 \end{bmatrix} + \begin{bmatrix} -4 & 7 \\ \frac{k}{3} & 5 \end{bmatrix} = \begin{bmatrix} -10 & 25 \\ -5 & 35 \end{bmatrix}$

$3h - 4 = -10$

$3h = -6$

$h = -2$

$-12 + \frac{k}{3} = -5$

$\frac{k}{3} = 7$

$k = 21$

(d) $3\begin{bmatrix} h & 4 \\ -5 & -k \end{bmatrix} + 2\begin{bmatrix} k & 4 \\ -2 & 3h \end{bmatrix} = \begin{bmatrix} 3 & 20 \\ -19 & 48 \end{bmatrix}$

$\begin{bmatrix} 3h & 12 \\ -15 & -3k \end{bmatrix} + \begin{bmatrix} 2k & 8 \\ -4 & 6h \end{bmatrix} = \begin{bmatrix} 3 & 20 \\ -19 & 48 \end{bmatrix}$

$3h + 2k = 3 \; \ldots (1)$

$-3k + 6h = 48$

$-k + 2h = 16 \; \ldots (2)$

$3h + 2k = 3 \; \ldots (1)$

(+) $4h - 2k = 32 \; \ldots (2) \times 2$

$7h = 35$

$h = 5$

From (1) :

$3(5) + 2k = 3$

$2k = -12$

$k = -6$

(e) $\frac{2}{3}\begin{bmatrix} -6 & 9 \\ h & k \end{bmatrix} + 3\begin{bmatrix} 4 & -1 \\ k & h \end{bmatrix} = \begin{bmatrix} 8 & 3 \\ 20 & 13 \end{bmatrix}$

$\begin{bmatrix} -4 & 6 \\ \frac{2}{3}h & \frac{2}{3}k \end{bmatrix} + \begin{bmatrix} 12 & -3 \\ 3k & 3h \end{bmatrix} = \begin{bmatrix} 8 & 3 \\ 20 & 13 \end{bmatrix}$

$\frac{2}{3}h + 3k = 20$

$2h + 9k = 60 \; \ldots (1)$

$\frac{2}{3}k + 3h = 13$

$2k + 9h = 39 \; \ldots (2)$

(1) $\times 9$: $2h + 9k = 60 \; \ldots (3)$

(2) $\times$ 2: $9h + 2k = 39 \; \ldots (4)$

$18h + 81k = 540 \; \ldots (3) \times 9$

(–) $18h + 4k = 78 \; \ldots (4) \times 2$

$77k = 462$

$k = 6$

From (1) : $2h + 9(6) = 60$

$2h = 6$

$h = 3$

14 $kQ + R = P$

$$\begin{bmatrix} \frac{1}{3}k & 2k \\ 3k & 5k \end{bmatrix} + \begin{bmatrix} p & 4 \\ 5 & 2q \end{bmatrix} = \begin{bmatrix} 6 & 10 \\ 14 & 7 \end{bmatrix}$$

$$\begin{aligned} 2k + 4 &= 10 \\ 2k &= 6 \\ k &= 3 \end{aligned}$$

$$\begin{aligned} 5k + 2q &= 7 \\ 5(3) + 2q &= 7 \\ 2q &= -8 \\ q &= -4 \end{aligned}$$

$$\begin{aligned} \frac{k}{3} + p &= 6 \\ \frac{3}{3} + p &= 6 \\ p &= 6 - 1 \\ p &= 5 \end{aligned}$$

15 (a) $3Y - \begin{bmatrix} 5 & 2 \\ 7 & -1 \end{bmatrix} = \begin{bmatrix} 9 & 5 \\ 14 & -20 \end{bmatrix} - 4Y$

$$7Y = \begin{bmatrix} 9 & 5 \\ 14 & -20 \end{bmatrix} + \begin{bmatrix} 5 & 2 \\ 7 & -1 \end{bmatrix}$$

$$7Y = \begin{bmatrix} 14 & 7 \\ 21 & -21 \end{bmatrix}$$

$$Y = \frac{1}{7}\begin{bmatrix} 14 & 7 \\ 21 & -21 \end{bmatrix}$$

$$Y = \begin{bmatrix} 2 & 1 \\ 3 & -3 \end{bmatrix}$$

(b) $\begin{bmatrix} 8 & 7 \\ 5 & -15 \end{bmatrix} - 3Y = \begin{bmatrix} -4 & -9 \\ 1 & 5 \end{bmatrix} + Y$

$$4Y = \begin{bmatrix} 8 & 7 \\ 5 & -15 \end{bmatrix} - \begin{bmatrix} -4 & -9 \\ 1 & 5 \end{bmatrix}$$

$$4Y = \begin{bmatrix} 12 & 16 \\ 4 & -20 \end{bmatrix}$$

$$Y = \begin{bmatrix} 3 & 4 \\ 1 & -5 \end{bmatrix}$$

(c) $2\begin{bmatrix} 1 & -1 \\ 8 & 3 \end{bmatrix} - 3Y = 2Y - 3\begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$

$$5Y = \begin{bmatrix} 2 & -2 \\ 16 & 6 \end{bmatrix} + \begin{bmatrix} 3 & -3 \\ -6 & 9 \end{bmatrix}$$

$$5Y = \begin{bmatrix} 5 & -5 \\ 10 & 15 \end{bmatrix}$$

$$Y = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

16 (a) $\begin{bmatrix} -7 & 5 & -2 \\ 3 & -6 & 9 \end{bmatrix} \begin{bmatrix} 10 & 2 \\ -4 & -7 \\ 9 & 0 \end{bmatrix}$

2×3 3×2

=

Can be multiplied

(b) $\begin{bmatrix} 3 & 1 \\ -6 & -3 \end{bmatrix} \begin{bmatrix} -8 & 4 & -1 \\ 2 & -5 & 6 \end{bmatrix}$

2×2 2×3

=

Can be multiplied

(c) $\begin{bmatrix} 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & -7 & -1 \\ -6 & -5 & 6 \end{bmatrix}$

1×2 2×3

=

Can be multiplied

(d) $\begin{bmatrix} -8 & 6 & -1 \\ 3 & -5 & 7 \end{bmatrix} \begin{bmatrix} 2 & -3 & 4 \end{bmatrix}$

2×3 1×3

≠

Cannot be multiplied

17 (a) $\begin{bmatrix} 1 & -4 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \\ -8 \end{bmatrix}$

1×3 3×1

Order = 1×1

(b) $\begin{bmatrix} 5 \\ 2 \\ -3 \end{bmatrix} \begin{bmatrix} -1 & -3 & 4 \end{bmatrix}$

3×1 1×3

Order = 3×3

(c) $\begin{bmatrix} 1 & 4 & -5 \\ -2 & 3 & -8 \end{bmatrix} \begin{bmatrix} 0 & 4 \\ -9 & 6 \\ -2 & -3 \end{bmatrix}$

2×3 3×2

Order = 2×2

18 (a) $\begin{bmatrix} 4 & 1 \\ 3 & 2 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -3 \end{bmatrix}$

$= \begin{bmatrix} 8-3 \\ 6-6 \\ -4+0 \end{bmatrix}$

$= \begin{bmatrix} 5 \\ 0 \\ -4 \end{bmatrix}$

(b) $\begin{bmatrix} -1 & 2 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 4 \\ -3 \end{bmatrix}$

$= \begin{bmatrix} -4-6 \\ -12-12 \end{bmatrix}$

$= \begin{bmatrix} -10 \\ -24 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & -4 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ -2 & -5 \end{bmatrix}$

$= \begin{bmatrix} 10 & 14 \end{bmatrix}$

(d) $\begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} -5 & 3 \\ 6 & -1 \end{bmatrix}$

$= \begin{bmatrix} -28 & 9 \\ 19 & -1 \end{bmatrix}$

(e) $\begin{bmatrix} -2 & -\frac{1}{5} & 3 \end{bmatrix} \begin{bmatrix} 3 \\ 15 \\ 4 \end{bmatrix}$

$= \begin{bmatrix} -6-3+12 \end{bmatrix}$

$= \begin{bmatrix} 3 \end{bmatrix}$

(f) $\begin{bmatrix} 5 & 2 \\ 1 & 4 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -3 & 3 \end{bmatrix}$

$= \begin{bmatrix} 4 & 11 \\ -10 & 13 \\ -4 & -2 \end{bmatrix}$

(g) $\begin{bmatrix} 1 & -2 & 5 \end{bmatrix} \begin{bmatrix} 1 & -5 \\ -4 & 6 \\ -3 & 4 \end{bmatrix}$

$= \begin{bmatrix} 1+8-15 & -5-12+20 \end{bmatrix}$

$= \begin{bmatrix} -6 & 3 \end{bmatrix}$

19 $\begin{bmatrix} x & 5 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ -x & 2 \end{bmatrix} = \begin{bmatrix} -7 & 10 \end{bmatrix}$

$\begin{bmatrix} 4x-5x & 10 \end{bmatrix} = \begin{bmatrix} -7 & 10 \end{bmatrix}$

$\begin{bmatrix} -x & 10 \end{bmatrix} = \begin{bmatrix} -7 & 10 \end{bmatrix}$

$-x = -7$

$x = 7$

20 $\begin{bmatrix} y & -4 \end{bmatrix} \begin{bmatrix} 0 & y \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} -12 & 20 \end{bmatrix}$

$y^2 + 4 = 20$

$y^2 = 16$

$y = \pm 4$

21 $\begin{bmatrix} z & 5 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ -z & 2 \end{bmatrix} = \begin{bmatrix} 12 & 10 \end{bmatrix}$

$\begin{bmatrix} 2z-5z & 10 \end{bmatrix} = \begin{bmatrix} 12 & 10 \end{bmatrix}$

$-3z = 12$

$z = -4$

22 $\begin{bmatrix} p & 3 \end{bmatrix} \begin{bmatrix} -3 & 1 \\ 2 & q \end{bmatrix} = \begin{bmatrix} -18 & 23 \end{bmatrix}$

$-3p + 6 = -18$

$-3p = -24$

$p = 8$

$p + 3q = 23$

$8 + 3q = 23$

$3q = 15$

$q = 5$

$p - q = 8 - 5 = 3$

23 $\begin{bmatrix} 3k & 4 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ -2k & 5 \end{bmatrix} = \begin{bmatrix} 12 & 20 \end{bmatrix}$

$-2k = 12$

$k = -6$

24 $\begin{bmatrix} 7 & x & 3x \end{bmatrix} \begin{bmatrix} x \\ -1 \\ -\frac{1}{3} \end{bmatrix} = \begin{bmatrix} 30 \end{bmatrix}$

$\begin{bmatrix} 7x - x - x \end{bmatrix} = \begin{bmatrix} 30 \end{bmatrix}$

$5x = 30$

$x = 6$

25 (a) $PI + Q$

$= P + Q$

$= \begin{bmatrix} 8 & -3 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} 7 & 1 \\ -5 & -6 \end{bmatrix}$

$= \begin{bmatrix} 15 & -2 \\ -1 & -1 \end{bmatrix}$

(b) $P - IQ$

$= P - Q$

$= \begin{bmatrix} 8 & -3 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 7 & 1 \\ -5 & -6 \end{bmatrix}$

$= \begin{bmatrix} 1 & -4 \\ 9 & 11 \end{bmatrix}$

26 (a) IAB

$= AB$

$= \begin{bmatrix} 5 & 2 & 4 \\ 6 & -3 & -7 \\ -1 & 8 & 3 \end{bmatrix} \begin{bmatrix} 1 & 6 & -2 \\ 5 & 7 & -3 \\ -7 & 4 & 9 \end{bmatrix}$

$= \begin{bmatrix} -13 & 60 & 20 \\ 40 & -13 & -66 \\ 18 & 62 & 5 \end{bmatrix}$

(b) $(A+B)I - I^2$

$= A + B - I$

$= \begin{bmatrix} 6 & 8 & 2 \\ 11 & 4 & -10 \\ -8 & 12 & 12 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 5 & 8 & 2 \\ 11 & 3 & -10 \\ -8 & 12 & 11 \end{bmatrix}$

27 (a) $BA = \begin{bmatrix} -1 & \frac{2}{3} \\ -1 & \frac{5}{3} \end{bmatrix} \begin{bmatrix} 5 & -2 \\ 9 & -3 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Hence, B is the inverse matrix of A.

(b) $QP = \begin{bmatrix} 4 & 5 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} -3 & -5 \\ 2 & 4 \end{bmatrix}$

$= \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$

Hence, Q is not the inverse matrix of P.

28 (a) $R^{-1} = \frac{1}{9-8}\begin{bmatrix} -3 & -2 \\ -4 & -3 \end{bmatrix} = \begin{bmatrix} -3 & -2 \\ -4 & -3 \end{bmatrix}$

(b) $S^{-1} = \frac{1}{9-10}\begin{bmatrix} 3 & -5 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} -3 & 5 \\ 2 & -3 \end{bmatrix}$

29 (a) $P^{-1} = \frac{1}{2}\begin{bmatrix} 3 & -4 \\ 2 & -2 \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & -2 \\ 1 & -1 \end{bmatrix}$

(b) $Q^{-1} = \frac{1}{4}\begin{bmatrix} 4 & 2 \\ -8 & -3 \end{bmatrix}$

$= \begin{bmatrix} 1 & \frac{1}{2} \\ -2 & -\frac{3}{4} \end{bmatrix}$

(c) $R^{-1} = \frac{1}{2}\begin{bmatrix} -5 & 4 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} -\frac{5}{2} & 2 \\ -\frac{3}{2} & 1 \end{bmatrix}$

(d) $S^{-1} = \frac{1}{-2}\begin{bmatrix} -2 & 4 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ \frac{1}{2} & -\frac{3}{2} \end{bmatrix}$

(e) $T^{-1} = \frac{1}{11}\begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} \frac{4}{11} & \frac{1}{11} \\ -\frac{3}{11} & \frac{2}{11} \end{bmatrix}$

(f) $U^{-1} = \frac{1}{4}\begin{bmatrix} -3 & 2 \\ -5 & 2 \end{bmatrix} = \begin{bmatrix} -\frac{3}{4} & \frac{1}{2} \\ -\frac{5}{4} & \frac{1}{2} \end{bmatrix}$

30 $\underbrace{\frac{1}{h}\begin{bmatrix} m & n \\ 9 & 3 \end{bmatrix}}_{A^{-1}} \underbrace{\begin{bmatrix} 3 & -1 \\ -9 & 4 \end{bmatrix}}_{A} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A^{-1} = \frac{1}{3}\begin{bmatrix} 4 & 1 \\ 9 & 3 \end{bmatrix}$

$\therefore h = 3, m = 4, n = 1$

31 (a) $4k - 24 = 0$

$k = 6$

(b) $-3k - 12 = 0$

$-3k = 12$

$k = -4$

(c) $6 + k = 0$

$k = -6$

(d) $-20 + 10k = 0$

$10k = 20$

$k = 2$

32 (a) $\begin{bmatrix} 2 & 3 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} 7 \\ 5 \end{bmatrix}$

(b) $7h - 2k = 24$

$3h - 4k = 26$

$\begin{bmatrix} 7 & -2 \\ 3 & -4 \end{bmatrix} \begin{bmatrix} h \\ k \end{bmatrix} = \begin{bmatrix} 24 \\ 26 \end{bmatrix}$

(c) $2x - 3y = -11$

$4x + y = 6$

$\begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -11 \\ 6 \end{bmatrix}$

33 (a) $\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-5}\begin{bmatrix} -2 & -1 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} -9 \\ 8 \end{bmatrix}$

$= \frac{1}{-5}\begin{bmatrix} 10 \\ 35 \end{bmatrix}$

$= \begin{bmatrix} -2 \\ -7 \end{bmatrix}$

(b) $\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-7}\begin{bmatrix} -1 & -2 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ 16 \end{bmatrix}$

$= \frac{1}{-7}\begin{bmatrix} -35 \\ 42 \end{bmatrix}$

$= \begin{bmatrix} 5 \\ -6 \end{bmatrix}$

(c) $\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 3 & -9 \\ 2 & -5 \end{bmatrix}\begin{bmatrix} 21 \\ 6 \end{bmatrix}$

$= \frac{1}{3}\begin{bmatrix} 9 \\ 12 \end{bmatrix}$

$= \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

34 (a) $\begin{bmatrix} 1 & -3 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} h \\ k \end{bmatrix} = \begin{bmatrix} 3 \\ 11 \end{bmatrix}$

$\begin{bmatrix} h \\ k \end{bmatrix} = \frac{1}{-1-(-6)}\begin{bmatrix} -1 & 3 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 11 \end{bmatrix}$

$= \frac{1}{5}\begin{bmatrix} 30 \\ 5 \end{bmatrix}$

$= \begin{bmatrix} 6 \\ 1 \end{bmatrix}$

$\therefore h = 6, k = 1$

(b) $\begin{bmatrix} 3 & 1 \\ 3 & -4 \end{bmatrix}\begin{bmatrix} m \\ n \end{bmatrix} = \begin{bmatrix} 9 \\ -6 \end{bmatrix}$

$\begin{bmatrix} m \\ n \end{bmatrix} = \frac{1}{-12-3}\begin{bmatrix} -4 & -1 \\ -3 & 3 \end{bmatrix}\begin{bmatrix} 9 \\ -6 \end{bmatrix}$

$= \frac{1}{-15}\begin{bmatrix} -4 & -1 \\ -3 & 3 \end{bmatrix}\begin{bmatrix} 9 \\ -6 \end{bmatrix}$

$= \frac{1}{-15}\begin{bmatrix} -30 \\ -45 \end{bmatrix}$

$= \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$\therefore m = 2, n = 3$

(c) $\begin{bmatrix} 2 & 1 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} 7 \\ -7 \end{bmatrix}$

$\begin{bmatrix} p \\ q \end{bmatrix} = \frac{1}{-7}\begin{bmatrix} -3 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 7 \\ -7 \end{bmatrix}$

$= \frac{1}{-7}\begin{bmatrix} -14 \\ -21 \end{bmatrix}$

$= \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$\therefore p = 2, q = 3$

35 (a) $\frac{1}{m}\begin{bmatrix} -4 & n \\ -1 & k \end{bmatrix}\begin{bmatrix} 5 & 2 \\ 1 & -4 \end{bmatrix} = I$

$\frac{1}{m}\begin{bmatrix} -4 & n \\ -1 & k \end{bmatrix} = \frac{1}{-18}\begin{bmatrix} -4 & -2 \\ -1 & 5 \end{bmatrix}$

$m = -22, n = -2, k = 5$

(b) $\begin{bmatrix} 5 & 2 \\ 1 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-22}\begin{bmatrix} -4 & -2 \\ -1 & 5 \end{bmatrix}\begin{bmatrix} 8 \\ 6 \end{bmatrix}$

$= \frac{1}{-22}\begin{bmatrix} -44 \\ 22 \end{bmatrix}$

$= \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

$\therefore x = 2, y = -1$

36 (a) $P = \frac{1}{2}\begin{bmatrix} -4 & 3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -2 & \frac{3}{2} \\ -1 & \frac{1}{2} \end{bmatrix}$

(b) $\begin{bmatrix} 1 & -3 \\ 2 & -4 \end{bmatrix}\begin{bmatrix} h \\ k \end{bmatrix} = \begin{bmatrix} 13 \\ 20 \end{bmatrix}$

$\begin{bmatrix} h \\ k \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -4 & 3 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 13 \\ 20 \end{bmatrix}$

$= \frac{1}{2}\begin{bmatrix} 8 \\ -6 \end{bmatrix}$

$= \begin{bmatrix} 4 \\ -3 \end{bmatrix}$

$\therefore h = 4, k = -3$

37 $9x + 6y = 21 \Rightarrow 3x + 2y = 7$

$8x + 6y = 20 \Rightarrow 4x + 3y = 10$

$\begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 10 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{1}\begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix}\begin{bmatrix} 7 \\ 10 \end{bmatrix}$

$= \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$\therefore x = 1, y = 2$

Hence, the price of a bottle of 500 m*l* mineral water and the price of a 1 000 m*l* mineral water are RM1 and RM2 respectively.

38 $6x+7y=138$

$8x+9y=182$

$$\begin{bmatrix}6 & 7\\8 & 9\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}138\\182\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{-2}\begin{bmatrix}9 & -7\\-8 & 6\end{bmatrix}\begin{bmatrix}138\\182\end{bmatrix}$$

$$=\frac{1}{-2}\begin{bmatrix}-32\\-12\end{bmatrix}$$

$$=\begin{bmatrix}16\\6\end{bmatrix}$$

$\therefore x=16,\ y=6$

Hence, the price of 1 kg of garlic and 1 kg of onions are RM16 and RM6 respectively.

Summative Practice 2

Multiple-Choice Questions

1 $\begin{bmatrix}5n\\-6\end{bmatrix}-\begin{bmatrix}3\\-m\end{bmatrix}=\begin{bmatrix}32\\-2\end{bmatrix}$

$$\begin{bmatrix}5n-3\\-6+m\end{bmatrix}=\begin{bmatrix}32\\-2\end{bmatrix}$$

$5n=35$

$n=7$

$-6+m=-2$

$m=4$

Answer: A

2 $\begin{bmatrix}4 & 2\\5 & -6\end{bmatrix}=\begin{bmatrix}a & 2\\5 & 2b\end{bmatrix}$

$a=4, b=-3$

$a-b=4-(-3)=7$

Answer: D

3 $\begin{bmatrix}1\\2\end{bmatrix}\begin{bmatrix}3 & 5\end{bmatrix}=\begin{bmatrix}3 & 5\\6 & 10\end{bmatrix}$

Answer: C

4 Determinant = $1(4)-2(3)=-2$

Answer: A

5 $\begin{bmatrix}2 & 1\\3 & -2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}4\\-1\end{bmatrix}$

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{-7}\begin{bmatrix}-2 & -1\\-3 & 2\end{bmatrix}\begin{bmatrix}4\\-1\end{bmatrix}$$

$$=\frac{1}{-7}\begin{bmatrix}-7\\-14\end{bmatrix}$$

$$=\begin{bmatrix}1\\2\end{bmatrix}$$

$\therefore x=1,\ y=2$

Answer: B

Structured Questions

1 $\begin{bmatrix}m & 1\\-4 & 3\end{bmatrix}-\frac{1}{3}\begin{bmatrix}12 & -15\\9 & n\end{bmatrix}=\begin{bmatrix}5 & 6\\-7 & -3\end{bmatrix}$

$m-4=5$

$3-\frac{1}{3}n=-2$

$-\frac{1}{3}n=-5$

$n=15$

$m+n=24$

2 $2\begin{bmatrix}-1 & p\\2 & 3\end{bmatrix}-3\begin{bmatrix}2 & 3\\-3 & 0\end{bmatrix}=\begin{bmatrix}-8 & -3\\q & 6\end{bmatrix}$

$2p-9=-3$

$p=3$

$4+9=q$

$q=13$

3 $\begin{bmatrix}3 & -4\\5 & 6\end{bmatrix}-\begin{bmatrix}3 & -8\\-3 & -2\end{bmatrix}=4\begin{bmatrix}0 & 1\\2 & q\end{bmatrix}$

$6+2=4q$

$4q=8$

$q=2$

4 $\begin{bmatrix}-2 & 4\\3 & 2\end{bmatrix}-P=\begin{bmatrix}-3 & 5\\2 & -4\end{bmatrix}$

$$P=\begin{bmatrix}-2 & 4\\3 & 2\end{bmatrix}-\begin{bmatrix}-3 & 5\\2 & -4\end{bmatrix}$$

$$P=\begin{bmatrix}1 & -1\\1 & 6\end{bmatrix}$$

5 $\begin{bmatrix}3 & -2 & 1\end{bmatrix}\begin{bmatrix}2 & -1\\-4 & 1\\-3 & 6\end{bmatrix}$

$=\begin{bmatrix}11 & 1\end{bmatrix}$

6 $\begin{bmatrix} 7 & 2x & 3 \end{bmatrix}\begin{bmatrix} x \\ -1 \\ -3 \end{bmatrix} = \begin{bmatrix} 6 \end{bmatrix}$

$$7x - 2x - 9 = 6$$
$$5x = 15$$
$$x = 3$$

7 The inverse matrix of $\begin{bmatrix} 2 & -4 \\ -1 & 3 \end{bmatrix}$

$= \frac{1}{2}\begin{bmatrix} 2 & -4 \\ -1 & 3 \end{bmatrix}$.

$= \begin{bmatrix} 1 & -2 \\ -\frac{1}{2} & \frac{3}{2} \end{bmatrix}$

But it is given that the inverse matrix

$= \begin{bmatrix} 1 & -2 \\ a & b \end{bmatrix}$.

By comparison, $a = -\frac{1}{2}$ and $b = \frac{3}{2}$.

$$\begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 2 & -4 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 2 \\ 4 \end{bmatrix}$$
$$= \frac{1}{2}\begin{bmatrix} 4-16 \\ -2+12 \end{bmatrix}$$
$$= \frac{1}{2}\begin{bmatrix} -12 \\ 10 \end{bmatrix}$$
$$x = -6,\ y = 5$$

8 (a) Determinant = 0

$$-6 + 3h = 0$$
$$3h = 6$$
$$h = 2$$

(b) $A = \begin{bmatrix} 1 & -3 \\ 5 & -6 \end{bmatrix}$

$$B = A^{-1} = \frac{1}{9}\begin{bmatrix} -6 & 3 \\ -5 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} -\frac{2}{3} & \frac{1}{3} \\ -\frac{5}{9} & \frac{1}{9} \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3 \\ 5 & -6 \end{bmatrix}\begin{bmatrix} m \\ n \end{bmatrix} = \begin{bmatrix} 3 \\ 18 \end{bmatrix}$$
$$\begin{bmatrix} m \\ n \end{bmatrix} = \frac{1}{9}\begin{bmatrix} -6 & 3 \\ -5 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 18 \end{bmatrix}$$
$$= \frac{1}{9}\begin{bmatrix} -18+54 \\ -15+18 \end{bmatrix}$$
$$= \frac{1}{9}\begin{bmatrix} 36 \\ 3 \end{bmatrix}$$
$$= \begin{bmatrix} 4 \\ \frac{1}{3} \end{bmatrix}$$

$\therefore m = 4,\ m = \frac{1}{3}$

8 (a) Determinant = 0

$$-6 + 3h = 0$$
$$3h = 6$$
$$h = 2$$

(b) $\begin{bmatrix} 1 & -3 \\ 5 & -6 \end{bmatrix}\begin{bmatrix} m \\ n \end{bmatrix} = \begin{bmatrix} 3 \\ 18 \end{bmatrix}$

(i) $B = A^{-1} = \frac{1}{9}\begin{bmatrix} -6 & 3 \\ -5 & 1 \end{bmatrix}$

(ii) $\begin{bmatrix} m \\ n \end{bmatrix} = \frac{1}{9}\begin{bmatrix} -6 & 3 \\ -5 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 18 \end{bmatrix}$

$$= \frac{1}{9}\begin{bmatrix} 36 \\ 3 \end{bmatrix}$$
$$= \begin{bmatrix} 4 \\ \frac{1}{3} \end{bmatrix}$$

$\therefore m = 4,\ n = \frac{1}{3}$

9 (a) $Q = P^{-1}$

$$Q = \frac{1}{15}\begin{bmatrix} 3 & 2 \\ -6 & 1 \end{bmatrix}$$

But it is given that $Q = \frac{1}{m}\begin{bmatrix} 3 & h \\ -6 & k \end{bmatrix}$

By comparison,

$m = 15,\ h = 2,\ k = 1$

(b) $\begin{bmatrix} 1 & -2 \\ 3 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -4 \\ -9 \end{bmatrix}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{15}\begin{bmatrix} 3 & 2 \\ -6 & 1 \end{bmatrix}\begin{bmatrix} -4 \\ -9 \end{bmatrix}$$
$$= \frac{1}{15}\begin{bmatrix} -30 \\ 15 \end{bmatrix}$$
$$= \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$\therefore x = -2,\ y = 1$

10 (a) $\frac{1}{m}\begin{bmatrix} h & k \\ 3 & 4 \end{bmatrix}$ is the inverse matrix of

$\begin{bmatrix} 5 & -2 \\ 3 & 4 \end{bmatrix}$.

But it is given that the inverse matrix is

$\frac{1}{20+6}\begin{bmatrix} 4 & 2 \\ -3 & 5 \end{bmatrix}$.

By comparison,

$m = 26, h = 4, k = 2$.

(b) $\begin{bmatrix} 5 & -2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 22 \\ 0 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{26}\begin{bmatrix} 4 & 2 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} 22 \\ 8 \end{bmatrix}$

$= \frac{1}{26}\begin{bmatrix} 104 \\ -26 \end{bmatrix}$

$= \begin{pmatrix} 4 \\ -1 \end{pmatrix}$

$\therefore x = 4, y = -1$

11 (a) $P = \frac{1}{-10+12}\begin{bmatrix} -5 & 4 \\ -3 & 2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -5 & 4 \\ -3 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & -4 \\ 3 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 14 \\ 19 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -5 & 4 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 14 \\ 19 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 6 \\ -4 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

$\therefore x = 3, y = -2$

12 (a) Inverse matrix

$= \frac{1}{-18+15}\begin{bmatrix} -9 & 3 \\ -5 & 2 \end{bmatrix}$

$= \frac{1}{-3}\begin{bmatrix} -9 & 3 \\ -5 & 2 \end{bmatrix}$

But it is given that the inverse matrix

$= \frac{1}{m}\begin{bmatrix} -9 & 3 \\ n & 2 \end{bmatrix}$.

By comparison,

$m = -3, \ n = -5$

(b) $\begin{bmatrix} 2 & -3 \\ 5 & -9 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 13 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-3}\begin{bmatrix} -9 & 3 \\ -5 & 2 \end{bmatrix}\begin{bmatrix} 7 \\ 13 \end{bmatrix}$

$= \frac{1}{-3}\begin{bmatrix} -24 \\ -9 \end{bmatrix}$

$= \begin{bmatrix} 8 \\ 3 \end{bmatrix}$

$\therefore x = 8, y = 3$

13 (a) Inverse matrix = $\frac{1}{4n-6}\begin{bmatrix} 4 & 1 \\ 6 & n \end{bmatrix}$

But it is given that the inverse matrix

$= \frac{1}{2}\begin{bmatrix} 4 & m \\ 6 & n \end{bmatrix}$

By comparison,

$= \ m = 1, 4n - 6 = 2 \Rightarrow n = 2$

(b) $\begin{bmatrix} 2 & -1 \\ 4 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -8 \\ -10 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-2}\begin{bmatrix} -3 & 1 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} -8 \\ -10 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-2}\begin{bmatrix} 14 \\ 12 \end{bmatrix} = \begin{bmatrix} -7 \\ -6 \end{bmatrix}$

$\therefore x = -7, y = -6$

14 (a) $P^{-1} = \frac{1}{2}\begin{bmatrix} -8 & 3 \\ -6 & 2 \end{bmatrix} = \begin{bmatrix} -4 & \frac{3}{2} \\ -3 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & -3 \\ 6 & -8 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6 \\ -15 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -8 & 3 \\ -6 & 2 \end{bmatrix}\begin{bmatrix} -6 \\ -15 \end{bmatrix}$

$= \frac{1}{2}\begin{bmatrix} 3 \\ 6 \end{bmatrix}$

$= \begin{bmatrix} 1.5 \\ 3 \end{bmatrix}$

$\therefore x = 1.5, y = 3$

15 (a) $P^{-1} = \frac{1}{-2}\begin{bmatrix} -3 & 4 \\ -1 & 2 \end{bmatrix}, \ h = -\frac{1}{2}, k = 4$

(b) $\begin{bmatrix} 2 & -4 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-2}\begin{bmatrix} -3 & 4 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 5 \\ 3 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-2}\begin{bmatrix} -3 \\ 1 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1.5 \\ -0.5 \end{bmatrix}$

$\therefore x = 1.5, y = -0.5$

16 $6x+3y=84$

$2x+y=28 \quad \ldots (1)$

$7x+4y=108 \quad \ldots (2)$

Inverse matrix of $\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$

$= \frac{1}{1}\begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$

$\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 28 \\ 108 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}\begin{bmatrix} 28 \\ 108 \end{bmatrix}$

$= \begin{bmatrix} 4 \\ 20 \end{bmatrix}$

$\therefore x=4,\ y=20$

Hence, the prices of a pineapple and a watermelon are RM4 and RM20 respectively.

17 $5x+6y=16 \quad \ldots (1)$

$2x+y=5 \quad \ldots (2)$

The inverse matrix of $\begin{bmatrix} 5 & 6 \\ 2 & 1 \end{bmatrix}$ is

$\frac{1}{-7}\begin{bmatrix} 1 & -6 \\ -2 & 5 \end{bmatrix}$.

$\begin{bmatrix} 5 & 6 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 16 \\ 5 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-7}\begin{bmatrix} 1 & =6 \\ -2 & 5 \end{bmatrix}\begin{bmatrix} 16 \\ 5 \end{bmatrix}$

$= \frac{1}{-7}\begin{bmatrix} -14 \\ -7 \end{bmatrix}$

$= \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$\therefore x=2,\ y=1$

Hence, the price of a 2B pencil and a ruler are RM2.00 and RM1.00 respectively.

18 $4x+2y=22 \Rightarrow 2x+y=11 \quad \ldots (1)$

$3x+2y=18 \quad \ldots (2)$

The inverse matrix of $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$ is

$\frac{1}{1}\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$

$\begin{bmatrix} 2 & -1 \\ =3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 11 \\ 18 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 11 \\ 18 \end{bmatrix}$

$= \begin{bmatrix} 4 \\ 3 \end{bmatrix}$

$\therefore x=4,\ y=3$

Hence, the prices of a red tilapia fish and a black tilapia fish are RM4 and RM3 respectively.

SPM SPOT

1 $\frac{1}{3}\begin{bmatrix} \frac{3}{2}h & -3 \\ 9 & -12 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} k & 4 \\ -8 & h \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ -1 & -3 \end{bmatrix}$

$\begin{bmatrix} \frac{h}{2} & -1 \\ 3 & -4 \end{bmatrix} + \begin{bmatrix} \frac{k}{2} & 2 \\ -4 & \frac{h}{2} \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ -1 & -3 \end{bmatrix}$

Equating the a_{22} elements,

$-4+\frac{h}{2}=-3$

$\frac{h}{2}=4-3$

$h=2$

Equating the a_{11} elements,

$\frac{h}{2}+\frac{k}{2}=-3$

$\frac{2}{2}+\frac{k}{2}=-3$

$\frac{k}{2}=-4$

$k=-8$

Hence, $h+k=2+(-8)=-6$

Answer: A

2 $\begin{bmatrix}3\\r\end{bmatrix}\begin{bmatrix}s & -2\end{bmatrix}=\begin{bmatrix}6 & -6\\-8 & 8\end{bmatrix}$

$\begin{bmatrix}3s & -6\\rs & -2r\end{bmatrix}=\begin{bmatrix}6 & -6\\-8 & 8\end{bmatrix}$

Equating the a_{11} elements,

$3s = 6$

$s = 2$

Equating the a_{22} elements,

$-2r = 8$

$r = -4$

Hence, $s - r = 2 - (-4) = 6$

Answer: D

3 (a) $M\begin{bmatrix}4 & 3\\10 & 11\end{bmatrix}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

Let the matrix be *A*.

$M = A^{-1}$

$M = \dfrac{1}{4(11)-3(10)}\begin{bmatrix}11 & -3\\-10 & 4\end{bmatrix}$

$M = \dfrac{1}{14}\begin{bmatrix}11 & -3\\-10 & 4\end{bmatrix}$

(b) $8x + 6y = \dfrac{80}{100}\times 6\ 500$

$8x + 6y = 5\ 200$

$4x + 3y = 2\ 600\ \ldots (1)$

$10x + 11y = \dfrac{90}{100}\times 8\ 000$

$10x + 11y = 7\ 200\ \ldots (2)$

$\begin{bmatrix}4 & 3\\10 & 11\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}2\ 600\\7\ 200\end{bmatrix}$

$\begin{bmatrix}x\\y\end{bmatrix}=\dfrac{1}{14}\begin{bmatrix}11 & -3\\-10 & 4\end{bmatrix}\begin{bmatrix}2\ 600\\7\ 200\end{bmatrix}$

$=\dfrac{1}{14}\begin{bmatrix}11(2\ 600)-3(7\ 200)\\-10(2\ 600)+4(7\ 200)\end{bmatrix}$

$=\dfrac{1}{14}\begin{bmatrix}7\ 000\\2\ 800\end{bmatrix}$

$=\begin{bmatrix}500\\200\end{bmatrix}$

Hence, $x = 500$ and $y = 200$.