Form 5: Chapter 8
Mathematical Modeling
Fully-worked Solutions

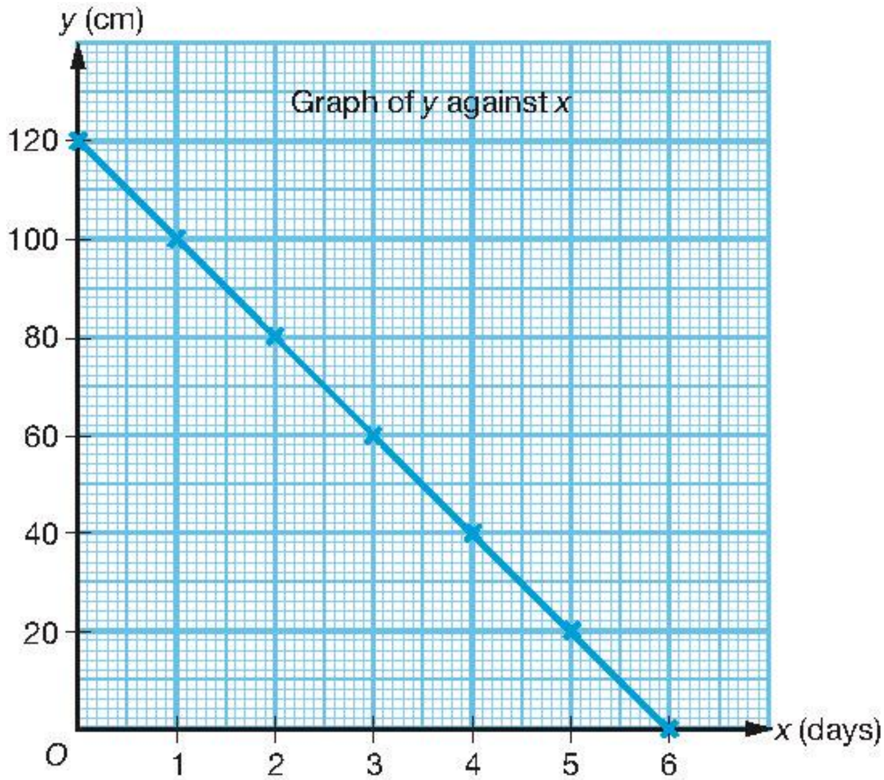
UPSKILL 8.1

1 (a) x represents the number of days and y represents the height of the snow (cm).

(b)

Number of days (x)	0	1	2	3	4	5	6
Height of snow (y cm)	120	100	80	60	40	20	0

(c)



(d) Gradient $= -\dfrac{120}{6} = -20$

y-intercept = 120

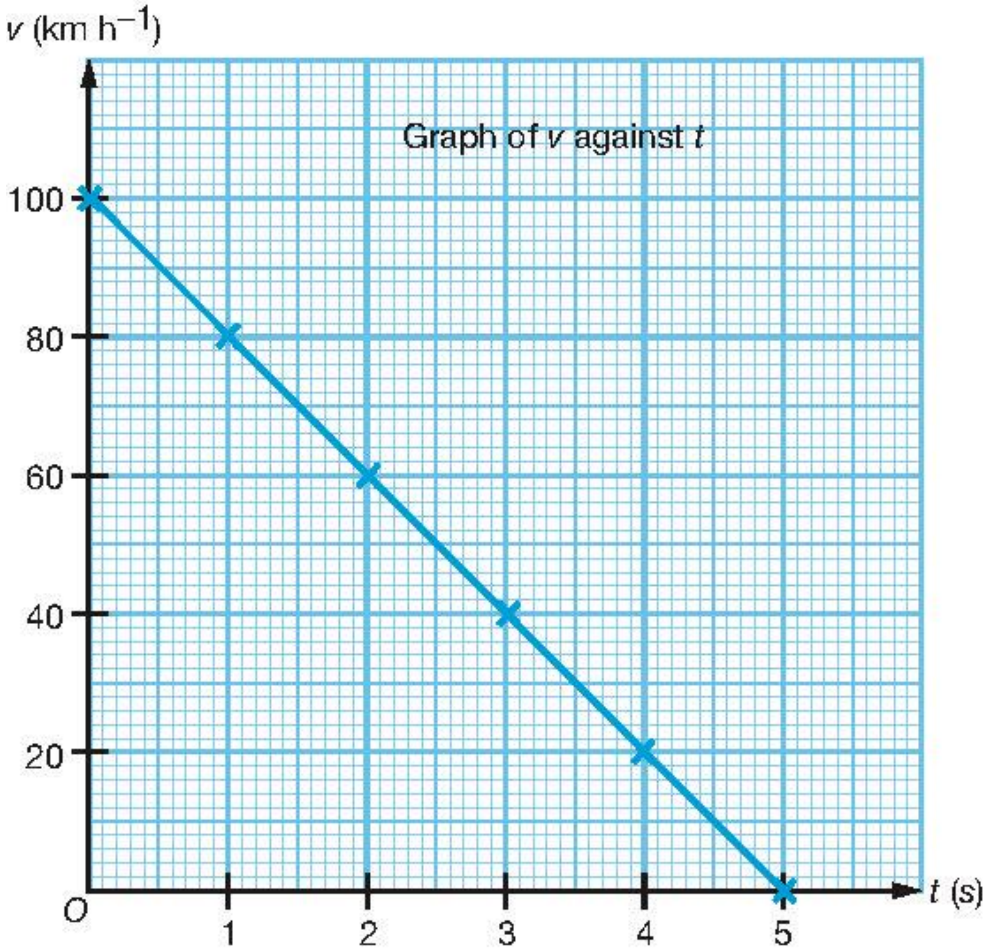
$\therefore\ y = -20x + 120$

2 (a) v represents speed, in km h^{-1} and t represents time, in seconds.

(b)

Speed (v km h^{-1})	100	80	60	40	20	0
Time (t s)	0	1	2	3	4	5

(c)



(d) Gradient $= -\frac{100}{5} - 20$

v- intercept = 100

$\therefore v = -20t + 100$

3 (a) The height of the spear when it is initially thrown

(b) 2 m

(c) Maximum horizontal distance of the spear

(d) $y = 0$

$$-\frac{7}{900}x^2 + \frac{13}{30}x + 2 = 0$$
$$-7x^2 + 390x + 1\,800 = 0$$
$$7x^2 - 390x - 1\,800 = 0$$
$$(x - 60)(7x + 30) = 0$$
$$x = 60 \text{ or } x = -\frac{30}{7}$$

$x = -\frac{30}{7}$ is not accepted.

$\therefore x = 60$

The maximum horizontal distance is 60 m.

4 (a) The length of the bridge (PQ)

(b) $h(x) = -\frac{x^2}{60} + 2x$

When $h(x) = 0$,

$$-\frac{x^2}{60} + 2x = 0$$
$$-x^2 + 120x = 0$$
$$-x(x - 120) = 0$$
$$x = 120$$

Distance between each rods = $\frac{120}{10} = 12$ m

(c) When $h(x) = 60$,

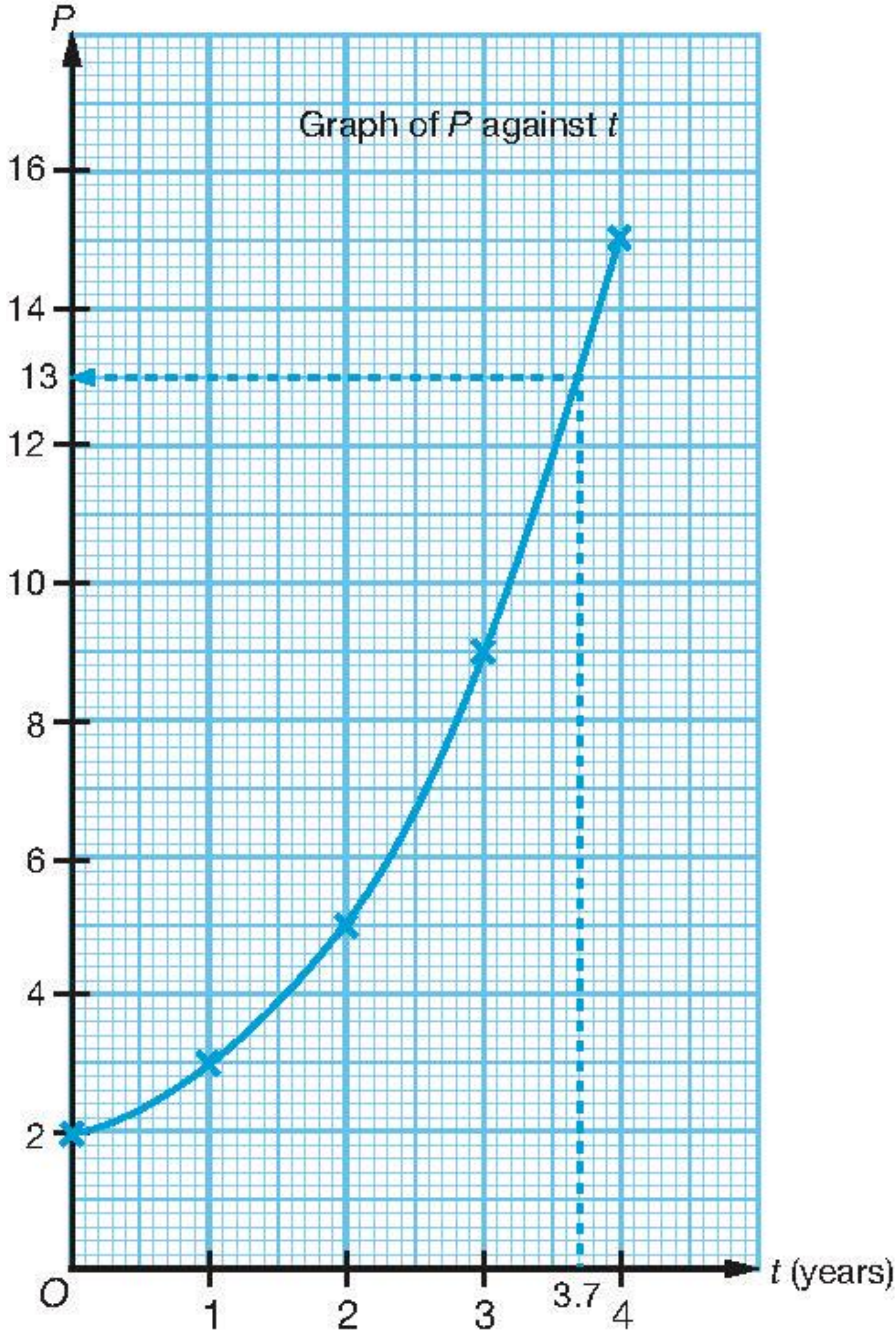
$$-\frac{x^2}{60} + 2x = 60$$
$$-x^2 + 120x = 3\,600$$
$$-x^2 + 120x - 3\,600 = 0$$
$$x^2 - 120x + 3\,600 = 0$$
$$(x - 60)(x - 60) = 0$$
$$x = 60$$

Hence, the horizontal distance of the concrete rod from P is 60 m.

5 (a)

t	0	1	2	3	4
P	2	3	5	9	15

(b)



(c) 13 panda bears

6 (a) $P(t) = ab^t$
$P(0) = ab^0 = 20$
$a = 20$

The number of monkeys increases 20% a year, i.e. $\frac{100+20}{100} \times 20 = 24$.

$P(1) = ab^1 = 24$
$20b = 24$
$b = 1.2$

(b) $P(t) = ab^t$
$P(10) = 20(1.2)^{10} = 124$ monkeys

Summative Practice 8

Multiple-Choice Questions

1 $f(x) = 200 + 50x$
Answer: C

2 $f(x) = 300 + 10x$
Answer: D

3 When $f(x) = 0$,
$-\frac{1}{20}x(x-20) = 0$
$x = 0$ or $x = 20$
$x = 0$ is not accepted.
$\therefore x = 20$
The midpoint is such that
$x = \frac{1}{2}(20) = 10$
When $x = 10$,
$f(20) = -\frac{1}{20}(10)(10-20)$
$= 5$
Hence, the maximum height of the motion of the ball is 5 m.
Answer: A

4 $J(x) = -\frac{7}{242}x^2 + \frac{13}{22}x + 1$
When $J(x) = 0$,
$-\frac{7}{242}x^2 + \frac{13}{22}x + 1 = 0$
$-7x^2 + 143x + 242 = 0$
$7x^2 - 143x - 242 = 0$
$(x-22)(7x+11) = 0$
$x = 22$ or $x = -\frac{11}{7}$
$x = -\frac{11}{7}$ is not accepted.
$\therefore x = 22$
Hence, the distance achieved by Johari ialah 22 m.
Answer: C

5 The number of students after 5 years
$= f(5)$
$= 100(0.95)^5$
$= 77$
Answer: A

6 The value of the car after 6 years
$= 120\ 000(0.95)^6$
= RM88 211
Answer: A

Structured Questions

1 $s(t) = ht + k$
$s(10) = 11$
$10h + k = 11 \ \dots (1)$

$s(15) = 8$
$15h + k = 8 \ \dots (2)$

$(1) - (2)$:
$-5h = 3$
$h = -\frac{3}{5}$

Substitute $h = -\frac{3}{5}$ into (1) :
$10\left(-\frac{3}{5}\right) + k = 11$
$-6 + k = 11$
$k = 17$

2 (a) x represents the number of photographs printed and y represents the payment, is RM.
(b) $y = 0.70x - 5$

3 (a) $h(x) = ax^2 + bx + c$

When $x = 0$, $h(0) = a(0)^2 + b(0) + c$
Thus, $c = 0$

When $x = 120$, $h(120) = 0$
$a(120)^2 + b(120) = 0$
$120a + b = 0 \ \dots (1)$

At the middle of the bridge,
$h(60) = 70$
$a(60)^2 + b(60) = 70$
$3\ 600a + 60b = 70$
$360a + 6b = 7 \ \dots (2)$

$720a + 6b = 0 \ \dots (1) \times 6$
(–) $360a + 6b = 7 \ \dots (2)$

$360a = -7$
$a = -\frac{7}{360}$

From (2) : $360\left(-\dfrac{7}{360}\right) + 6b = 7$

$$-7+6b=7$$

$$b=\frac{14}{6}$$

$$b=\frac{7}{3}$$

Hence, $a=-\dfrac{7}{360}$, $b = \dfrac{7}{3}$, $c = 0$

(b) $h(x) = -\dfrac{7}{360}x^2 + \dfrac{7}{3}x$

When $h(x) = 58\dfrac{4}{5}$,

$$-\frac{7}{360}x^2 + \frac{7}{3}x = \frac{294}{5}$$

$$-7x^2+840x-21\ 168 = 0$$

$$x^2-120x+3\ 024 = 0$$

$$(x-36)(x-84)=0$$

$$x=36 \text{ or } x=84$$

Hence, the horizontal distance from P is 36 m or 84 m.

4 (a) The object hits the surface of the sea.

(b) When $y = -64$,

$$24t - 4t^2 = -64$$

$$4t^2-24t-64=0$$

$$t^2-6t-16=0$$

$$(t-8)(t+2)=0$$

$t=8$ or $t=-2$

$t=-2$ is not accepted.

$\therefore t=8$

5 (a) $y=-\dfrac{3}{100}(x-50)^2+75$

When $x = 0$, $y=-\dfrac{3}{100}(0-50)^2+75$

$$y=75-75$$

$$y=0$$

(b) When $y = 0$, x is the distance of QE.

(c) $$-\frac{3}{100}(x-50)^2+75=0$$

$$-3(x-50)^2+7\ 500=0$$

$$-3\left(x^2-100x+2\ 500\right)+7\ 500=0$$

$$-3x^2+300x-7\ 500+7\ 500=0$$

$$-3x^2+300x=0$$

$$x^2-100x=0$$

$$x(x-100)=0$$

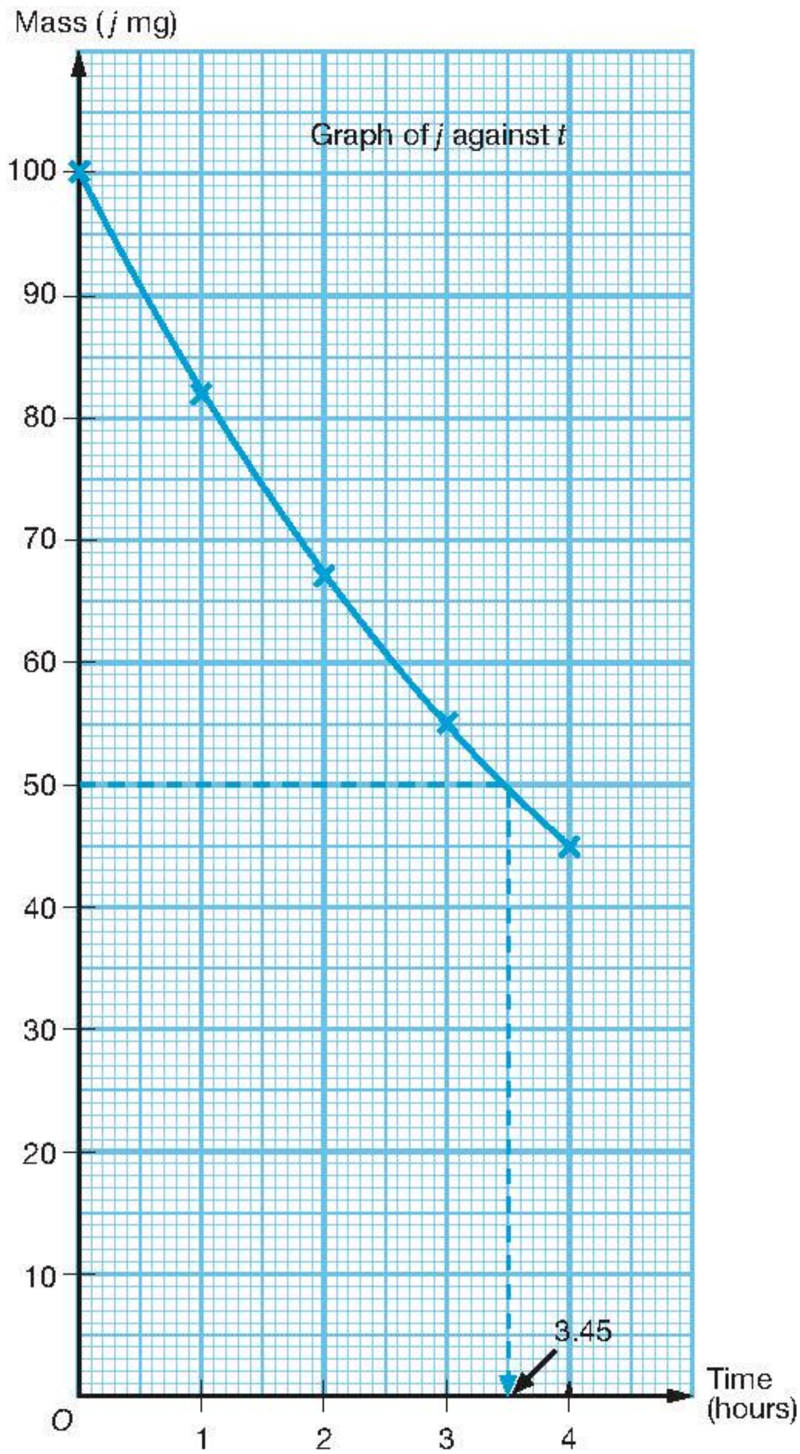
$$x=100$$

$$QE=100 \text{ cm}$$

6 (a)

Time (*t hours*)	0	1	2	3	4
Mass (*j* mg)	100	81.87	67.03	54.88	44.93

(b)



(c) 3.45 hours

7 (a) $y = a(b)^x$

When $x = 0$, $y = 5\ 000$

$5\ 000 = a(b)^0$

$a = 5\ 000$

When $x = 1$, $y = 5\ 200$

$5\ 200 = a(b)^1$

$ab = 5\ 200$

$5\ 000b = 5\ 200$

$b = 1.04$

(b) $y = 5\ 000\ (1.04)^x$

When $x = 2$,

$y = 5\ 000\ (1.04)^2$

$y = 5\ 408$

Puan Hani's saving is RM5 408.

8 (a) $n = ae^{2t}$

When $t = 0$, $n = 2$.

$2 = a\left[2.718^{2(0)}\right]$

$2 = a(1)$

$a = 2$

(b) $n = 2e^{2t}$

When $t = 4.25$,

$n = 2e^{2(4.25)} = 9\ 821$ bacteria

SPM SPOT

1 $f(x) = 50 + 2x$

Answer: D

2 (a) $P(x) = ar^{x-1}$

When $x = 1$, $P(1) = ar^{1-1} = 120$

$ar^0 = 120$

$a = 120$

$P(x) = 120r^{x-1}$

When $x = 2$, $P(2) = 118$

$120r^{2-1} = 118$

$120r = 118$

$r = \frac{118}{120}$

$r = \frac{59}{60}$

(b) $P(x) = 120\left(\frac{59}{60}\right)^{x-1} = 120\left(\frac{59}{60}\right)^{4-1} = 114$

Hence, the number of the Malayan tigers in the fourth year (2024) is 114.

